Blockchain-based Distributed Power Networks: A Mean-Field-Type Game Perspective

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Mean-Field Games: some references

- Infinite number of agents: Borel 1921, Volterra'26, Hotelling'29,von Neumann'44, Nash'51, Wardrop'52, Aumann'64, Selten'70, Schmeidler'73, Dubey et al.'80-, etc
- Discrete-time/state mean-field games:
  - Jovanovic'82, Jovanovic & Rosenthal'88, Bergins & Bernhardt'92, Weibull& Benaïm'03-, Weintraub, Benkard, Van Roy'05-, Sandholm '06-, Adlaska, Johari, Goldsmith'08-, Benaïm & Le Boudec'08-, Gast & Gaujal'09, Bardenave'09-, Gomes, Mohr & Souza'10-, Borkar & Sundaresan'12, Elliott'12-,Bayraktar, Budhiraja, Cohen'17- . . .
- Continuous-time mean-field games
  - Krusell & Smith'98, Benamou & Brenier'00-, Huang, Caines, Malhame'03-, Lasry & Lions'06-, Kotelenez & Kurtz'07-, Li & Zhang'08-, Buckdahn, Djehiche, Li and Peng'09-, Gueant'09-, Gomes et al.'09-, Yin, Mehta, Meyn, and Shanbhag'10, Djehiche et al' 10, Feng et al.'10-, Dogbe'10-, Achdou et al.'10-, LaChapelle'10-, Zhu, Başar'11, Bardi'12, Bensoussan,Sung,Yam,Yung'12-, Kolokoltsov'12-, Carmona & Delarue'12-,Yong'13-,Gangbo & Swiech'14-, Pham'16-,Fischer'17-,Nuno'17-, . . .

Simple Framework

- LQ game inspired.
- State driven by jump-diffusion- regime switching process of conditional mean-field type
- common noise which is driven by jump-diffusion- regime switching process of conditional mean-field type
- Conditional mean and (co-)variance
- Discount rate, coefficients are regime dependent
- State observation: $y = (x, B_o, N_o, s)$.
- Cost observation: noisy and noise driven by jump-diffusion-regime switching process of conditional mean-field type

Simple Framework

$$
\begin{aligned}
&x(0) = x_0, \quad s(0) = s_0,\\
&dx = bdt + \sigma dB + \int_\Theta \mu(\cdot,\theta)\tilde{N}(dt,d\theta) + \sigma_o dB_o + \int_\Theta \mu_o(\cdot,\theta)\tilde{N}_o(dt,d\theta),\\
&s(t) \text{ transition rate } \tilde{q}_{ss'}
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
b &= b_0 + b_1 x + \bar{b}_1\hat{x} + \sum_{j=1}^n b_{j2}u_j + \sum_{j=1}^n \bar{b}_{j2}\hat{u}_j,\\
\sigma &= \sigma_0 + \sigma_1 x + \bar{\sigma}_1\hat{x} + \sum_{j=1}^n \sigma_{j2}u_j + \sum_{j=1}^n \bar{\sigma}_{j2}\hat{u}_j,\\
\mu(\cdot,\theta) &= \mu_0 + \mu_1 x + \bar{\mu}_1\hat{x} + \sum_{j=1}^n \mu_{j2}u_j + \sum_{j=1}^n \bar{\mu}_{j2}\hat{u}_j,\\
\sigma_o &= \sigma_{o0} + \sigma_{o1} x + \bar{\sigma}_{o1}\hat{x} + \sum_{j=1}^n \sigma_{oj2}u_j + \sum_{j=1}^n \bar{\sigma}_{oj2}\hat{u}_j,\\
\mu_o(\cdot,\theta) &= \mu_{o0} + \mu_{o1} x + \bar{\mu}_{o1}\hat{x} + \sum_{j=1}^n \mu_{oj2}u_j + \sum_{j=1}^n \bar{\mu}_{oj2}\hat{u}_j,
\end{aligned}
$$

$$b_k, \sigma_k, \bar{b}_k, \bar{\sigma}_k : \ [0,T] \times \mathcal{S} \to \mathbb{R}, \ k \in \{0,1\}$$
$$b_{j2}, \sigma_{j2}, \bar{b}_{j2}, \bar{\sigma}_{j2} : \ [0,T] \times \mathcal{S} \to \mathbb{R}, \ j \in \mathcal{I},$$
$$\mu_k, \mu_{ok}, \bar{\mu}_k, \bar{\mu}_{ok} : \ [0,T] \times \mathcal{S} \times \Theta \to \mathbb{R}, \ k \in \{0,1\}$$
$$\mu_{j2}, \mu_{oj2}, \bar{\mu}_{j2}, \bar{\mu}_{oj2} : \ [0,T] \times \mathcal{S} \times \Theta \to \mathbb{R}, \ j \in \mathcal{I},$$
$$\mathbb{F} := \mathbb{F}^{B,N,B_o,N_o,s},$$
$$\mathcal{U}_i = \{u_i(\cdot) \in \mathcal{L}^2_{\mathbb{F}}([0,T] \times \mathcal{S}; \mathbb{R}); \ u_i(.) \in U_i \ a.e.\, t \in [0,T], \ \mathbb{P} - a.s.\},$$
$$\hat{x}(t) := \mathbb{E}[x(t) | \ \mathcal{F}^{B_o,N_o,s}_{t-}],$$
$$\hat{u} := \mathbb{E}[x| \ \mathcal{F}^{B_o,N_o,s}]$$

Noisy cost measurement

$$
\begin{aligned}
L_i(x,s,u) = &\frac{1}{2}q_{iT}x^2(T) + \frac{1}{2}\bar{q}_{iT}\hat{x}^2(T) + \epsilon_{i3T}x(T) + \bar{\epsilon}_{i3T}\hat{x}(T)\\
&+\frac{1}{2}\int_0^T \left(q_i x^2 + \bar{q}_i \hat{x}^2 + r_i u_i^2 + \bar{r}_i \hat{u}_i^2 \right) dt\\
&+\int_0^T \left(\epsilon_{i1} x u_i + \bar{\epsilon}_{i1}\hat{x}\hat{u}_i + \epsilon_{i2}u_i + \bar{\epsilon}_{i2}\hat{u}_i + \epsilon_{i3}x + \bar{\epsilon}_{i3}\hat{x} \right) dt\\
&+\int_0^T \left(\xi_{i0,B_o} + \xi_{i1,B_o}\hat{x} + \frac{1}{2}\xi_{i2,B_o}x^2 + \frac{1}{2}\bar{\xi}_{i2,B_o}\hat{x}^2 \right) dB_o\\
&+\int_0^T \int_\Theta \left(\xi_{i0,N_o} + \xi_{i1,N_o}\hat{x} + \frac{1}{2}\xi_{i2,N_o}x^2 + \frac{1}{2}\bar{\xi}_{i2,N_o}\hat{x}^2 \right) \tilde{N}_o(dt,d\theta),
\end{aligned}
\tag{2}
$$

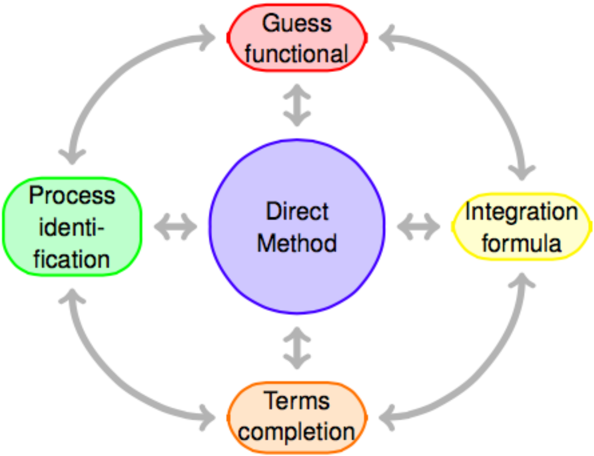
$$
\begin{aligned}
&q_{iT}, \bar{q}_{iT}, \epsilon_{i3T}, \epsilon_{i3T} \;:\; \mathcal{S} \to \mathbb{R},\\
&q_i, \bar{q}_i, r_i, \bar{r}_i, \epsilon_{ik}, \epsilon_{ik} \;:\; [0,T]\times\mathcal{S} \to \mathbb{R},\ k \in \{1,2,3\}\\
&\xi_{ik,B_o}, \bar{\xi}_{i2,B_o} \;:\; [0,T]\times\mathcal{S} \to \mathbb{R},\ k \in \{0,1,2\},\\
&\xi_{ik,N_o}, \bar{\xi}_{i2,N_o} \;:\; [0,T]\times\mathcal{S}\times\Theta \to \mathbb{R},\ k \in \{0,1,2\},
\end{aligned}
$$

Problem

Find $(x^*, u^*, \hat{x}^*, \hat{u}^*)$ such that

$$\left\{ \begin{array}{l} i \in \mathcal{I} := \{1, 2, \ldots, n\}, \\ \mathbb{E}\left[L_i(x, s, u^*) | x(0) = x_0, s(0) = s_0\right] = \inf_{u_i \in \mathcal{U}_i} \mathbb{E}\left[L_i(x, s, u_i, u^*_{-i}) \, | x(0) = x_0, s(0) = s_0\right], \end{array} \right. \tag{3}$$

Direct Method



Solution

The equilibrium strategy and the equilibrium cost are given by:

$$\begin{aligned} u_i^* &= \hat{u}_i^* - \frac{\tilde{\tau}_i}{c_i}(x - \hat{x}), \\ \hat{u}_i^* &= -\frac{\bar{\tau}_{i1}}{\bar{c}_i}\hat{x} - \frac{\bar{\tau}_{i0}}{\bar{c}_i}, \\ L_i^*(x, s, u^*) &= \frac{1}{2}\mathbb{E}[\alpha_i(0, s(0))(x_0 - \hat{x}_0)^2] + \frac{1}{2}\mathbb{E}[\beta_i(0, s(0))\hat{x}_0^2] \\ &\quad + \mathbb{E}[\gamma_i(0, s(0))\hat{x}_0] + \mathbb{E}[\delta_i(0, s(0))], \end{aligned} \tag{4}$$

$$
\begin{aligned}
d\alpha_i &= -\Big\{ 2b_1\alpha_i + \alpha_i[\sigma_1^2 + \sigma_{o1}^2 + \int_\Theta \mu_1^2\nu(d\theta) + \int_\Theta \mu_{o1}^2\nu_o(d\theta)] \\
&+q_i - \frac{\tilde{\tau}_i^2}{c_i} + \sum_{s'\neq s} \tilde{q}_{ss'}[\alpha_i(t,s') - \alpha_i(t,s)] + 2\alpha_{i,B_o}\sigma_{o1} + 2\int_\Theta \alpha_{i,N_o}\mu_{o1}\nu_o(d\theta) \\
&+\alpha_i[\sum_{j\neq i}\sigma_{j2}\frac{\tilde{\tau}_j}{c_j}]^2 + \alpha_i\int_\Theta[\sum_{j\neq i}\mu_{j2}\frac{\tilde{\tau}_j}{c_j}]^2\nu(d\theta) + \alpha_i[\sum_{j\neq i}\sigma_{oj2}\frac{\tilde{\tau}_j}{c_j}]^2 + \alpha_i\int_\Theta[\sum_{j\neq i}\mu_{oj2}\frac{\tilde{\tau}_j}{c_j}]^2\nu_o(d\theta) \\
&-2\alpha_i\sum_{j\neq i}\{b_{j2} + \sigma_1\sigma_{j2} + \int_\Theta \mu_1\mu_{j2}\nu(d\theta)\}\frac{\tilde{\tau}_j}{c_j} - 2\alpha_i\sum_{j\neq i}\{\sigma_{o1}\sigma_{oj2} + \int_\Theta \mu_{o1}\mu_{oj2}\nu_o(d\theta)\}\frac{\tilde{\tau}_j}{c_j} \\
&-2\sum_{j\neq i}\{\alpha_{i,B_o}\sigma_{oj2} + \int_\Theta \alpha_{i,N_o}\mu_{oj2}\nu_o(d\theta)\}\frac{\tilde{\tau}_j}{c_j}\Big\}\, dt \\
&-\left(\xi_{i2,B_o} + \alpha_i[2\sigma_{o1} - 2\sum_{j\neq i}\sigma_{oj2}\frac{\tilde{\tau}_j}{c_j}]\right) dB_o - \int_\Theta \left(\xi_{i2,N_o} + \alpha_i[2\mu_{o1} - 2\sum_{j\neq i}\mu_{oj2}\frac{\tilde{\tau}_j}{c_j}]\right)\tilde{N}_o(dt, d\theta
\\
\alpha_i(T,s) &= q_i(T,s), \\
\alpha_{i,B_o} &= -\left(\xi_{i2,B_o} + \alpha_i[2\sigma_{o1} - 2\sum_{j\neq i}\sigma_{oj2}\frac{\tilde{\tau}_j}{c_j}]\right), \\
\alpha_{i,N_o} &= -\left(\xi_{i2,N_o} + \alpha_i[2\mu_{o1} - 2\sum_{j\neq i}\mu_{oj2}\frac{\tilde{\tau}_j}{c_j}]\right),
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
d\beta_i = & -\Bigg\{ 2\hat{b}_1\beta_i + \beta_i[\hat{\sigma}_{o1}^2 + \int_\Theta \hat{\mu}_{o1}^2 \nu_o(d\theta)] + \hat{q}_i - \frac{\bar{\tau}_{i1}^2}{\bar{c}_i} + \sum_{s'\neq s} \tilde{q}_{ss'}[\beta_i(t,s') - \beta_i(t,s)] \\
& + \alpha_i[\hat{\sigma}_1^2 + \int_\Theta \hat{\mu}_1^2 \nu(d\theta)] + 2\beta_{i,B_o}\hat{\sigma}_{o1} + 2\int_\Theta \beta_{i,N_o}\hat{\mu}_{o1}\nu_o(d\theta) \\
& + \alpha_i[\sum_{j\neq i} \hat{\sigma}_{j2}\frac{\bar{\tau}_{j1}}{\bar{c}_j}]^2 + \alpha_i\int_\Theta [\sum_{j\neq i}\hat{\mu}_{j2}\frac{\bar{\tau}_{j1}}{\bar{c}_j}]^2\nu(d\theta) + \beta_i[\sum_{j\neq i}\hat{\sigma}_{oj2}\frac{\bar{\tau}_{j1}}{\bar{c}_j}]^2 + \beta_i\int_\Theta[\sum_{j\neq i}\hat{\mu}_{oj2}\frac{\bar{\tau}_{j1}}{\bar{c}_j}]^2\nu_o(d\theta) \\
& - 2\alpha_i\sum_{j\neq i}\{\hat{\sigma}_1\hat{\sigma}_{j2} + \int_\Theta \hat{\mu}_1\hat{\mu}_{j2}\nu(d\theta)\}\frac{\bar{\tau}_{j1}}{\bar{c}_j} - 2\beta_i\sum_{j\neq i}\{\hat{\sigma}_{o1}\hat{\sigma}_{oj2} + \int_\Theta \hat{\mu}_{o1}\hat{\mu}_{oj2}\nu_o(d\theta)\}\frac{\bar{\tau}_{j1}}{\bar{c}_j} \\
& - 2\beta_i\sum_{j\neq i}\hat{b}_{j2}\frac{\bar{\tau}_{j1}}{\bar{c}_j} - 2\sum_{j\neq i}\{\beta_{i,B_o}\hat{\sigma}_{oj2} + \int_\Theta \beta_{i,N_o}\hat{\mu}_{oj2}\}\frac{\bar{\tau}_{j1}}{\bar{c}_j}\Bigg\} dt \\
& - \left(\hat{\xi}_{i2,B_o} + \beta_i[2\hat{\sigma}_{o1} - 2\sum_{j\neq i}\hat{\sigma}_{oj2}\frac{\bar{\tau}_{j1}}{\bar{c}_j}]\right) dB_o - \int_\Theta \left(\hat{\xi}_{i2,N_o} + 2\beta_i[\hat{\mu}_{o1} - \sum_{j\neq i}\hat{\mu}_{oj2}\frac{\bar{\tau}_{j1}}{\bar{c}_j}]\right)\tilde{N}_o(dt, d
\end{aligned}
$$

$$
\beta_i(T,s) = q_i(T,s) + \bar{q}_i(T,s) = \hat{q}_i(T,s),\ \beta_{i,B_o} = -\left(\hat{\xi}_{i2,B_o} + \beta_i[2\hat{\sigma}_{o1} - 2\sum_{j\neq i}\hat{\sigma}_{oj2}\frac{\bar{\tau}_{j1}}{\bar{c}_j}]\right),
$$

$$
\beta_{i,N_o} = -\left(\hat{\xi}_{i2,N_o} + \beta_i[2\hat{\mu}_{o1} - 2\sum_{j\neq i}\hat{\mu}_{oj2}\frac{\bar{\tau}_{j1}}{\bar{c}_j}]\right), \quad (6)
$$

Let

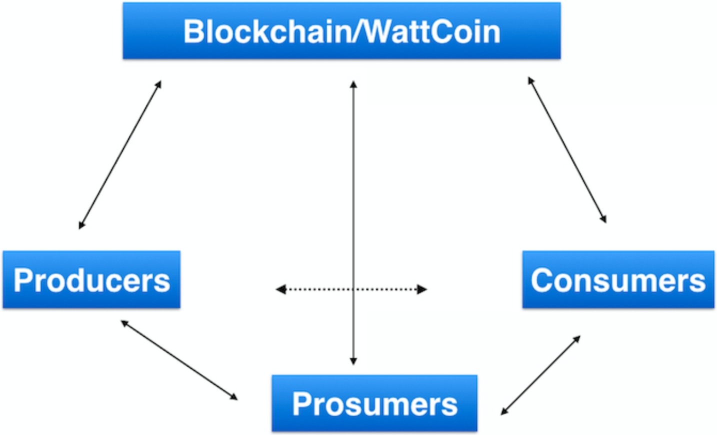
$$
\begin{aligned}
\Omega :=\; & \hat{b}_1\gamma_i + [\hat{\epsilon}_{i3} - \frac{\bar{\tau}_{i1}\bar{\tau}_{i0}}{\bar{c}_i}] + \sum_{s'\neq s} \tilde{q}_{ss'}[\gamma_i(t,s') - \gamma_i(t,s)] + \alpha_i[\sigma_0\hat{\sigma}_1 + \int_{\Theta} \mu_0\hat{\mu}_1\nu(d\theta)] \\
& +\beta_i[b_0 + \sigma_{o0}\hat{\sigma}_{o1} + \int_{\Theta} \mu_{o0}\hat{\mu}_{o1}\nu_o(d\theta)] + \beta_{i,B_o}\sigma_{o0} + \int_{\Theta} \beta_{i,N_o}\mu_{o0}\nu_o(d\theta) \\
& +\gamma_{i,B_o}\hat{\sigma}_{o1} + \int_{\Theta} \gamma_{i,B_o}\hat{\mu}_{o1}\nu_o(d\theta) \\
& +\alpha_i[\sum_{j\neq i}\hat{\sigma}_{j2}\frac{\bar{\tau}_{j0}}{\bar{c}_j}][\sum_{k\neq i}\hat{\sigma}_{k2}\frac{\bar{\tau}_{k1}}{\bar{c}_k}] + \alpha_i\int_{\Theta}[\sum_{j\neq i}\hat{\mu}_{j2}\frac{\bar{\tau}_{j0}}{\bar{c}_j}][\sum_{k\neq i}\hat{\mu}_{k2}\frac{\bar{\tau}_{k1}}{\bar{c}_k}]\nu(d\theta) \\
& +\beta_i[\sum_{j\neq i}\hat{\sigma}_{oj2}\frac{\bar{\tau}_{j0}}{\bar{c}_j}][\sum_{j\neq i}\hat{\sigma}_{oj2}\frac{\bar{\tau}_{j1}}{\bar{c}_j}] + \beta_i\int_{\Theta}[\sum_{j\neq i}\hat{\mu}_{oj2}\frac{\bar{\tau}_{j0}}{\bar{c}_j}][\sum_{j\neq i}\hat{\mu}_{oj2}\frac{\bar{\tau}_{j1}}{\bar{c}_j}]\nu_o(d\theta) \\
& -\alpha_i\sum_{j\neq i}\{\hat{\sigma}_1\hat{\sigma}_{j2} + \int_{\Theta}\hat{\mu}_1\hat{\mu}_{j2}\nu(d\theta)\}\frac{\bar{\tau}_{j0}}{\bar{c}_j} - \beta_i\sum_{j\neq i}\{\hat{\sigma}_{o1}\hat{\sigma}_{oj2} + \int_{\Theta}\hat{\mu}_{o1}\hat{\mu}_{oj2}\nu_o(d\theta)\}\frac{\bar{\tau}_{j0}}{\bar{c}_j} \\
& -\beta_i\sum_{j\neq i}\hat{b}_{j2}\frac{\bar{\tau}_{j0}}{\bar{c}_j} - \sum_{j\neq i}\{\beta_{i,B_o}\hat{\sigma}_{oj2} + \int_{\Theta}\beta_{i,N_o}\hat{\mu}_{oj2}\}\frac{\bar{\tau}_{j0}}{\bar{c}_j} - \alpha_i\sum_{j\neq i}\{\sigma_0\hat{\sigma}_{j2} + \int_{\Theta}\mu_0\hat{\mu}_{j2}\nu(d\theta)\}\frac{\bar{\tau}_{j1}}{\bar{c}_j} \\
& -\beta_i\sum_{j\neq i}\{\sigma_{o0}\hat{\sigma}_{oj2} + \int_{\Theta}\mu_{o0}\hat{\mu}_{oj2}\nu_o(d\theta)\}\frac{\bar{\tau}_{j1}}{\bar{c}_j} \\
& -\sum_{j\neq i}\{\gamma_{i,B_o}\hat{\sigma}_{oj2} + \int_{\Theta}\gamma_{i,N_o}(t,\theta)\hat{\mu}_{oj2}\nu_o(d\theta)\}\frac{\bar{\tau}_{j1}}{\bar{c}_j} - \gamma_i\sum_{j\neq i}\hat{b}_{j2}\frac{\bar{\tau}_{j1}}{\bar{c}_j}
\end{aligned}
$$

$$
\begin{aligned}
& d\gamma_i = -\Omega dt \\
& - \left[\xi_{i1,B_o} + \beta_i(\sigma_{o0} - \sum_{j\neq i} \hat{\sigma}_{oj2} \frac{\bar{\tau}_{j0}}{\bar{c}_j}) + \gamma_i(\hat{\sigma}_{o1} - \sum_{j\neq i} \hat{\sigma}_{oj2} \frac{\bar{\tau}_{j1}}{\bar{c}_j}) \right] dB_o \\
& - \int_{\Theta} \left[\xi_{i1,N_o} + \beta_i(\mu_{o0} - \sum_{j\neq i} \hat{\mu}_{oj2} \frac{\bar{\tau}_{j0}}{\bar{c}_j}) + \gamma_i(\hat{\mu}_{o1} - \sum_{j\neq i} \hat{\mu}_{oj2} \frac{\bar{\tau}_{j1}}{\bar{c}_j}) \right] \tilde{N}_o(dt, d\theta), \\
& \gamma_i(T,s) = \epsilon_{i3}(T,s) + \bar{\epsilon}_{i3}(T,s) =: \hat{\epsilon}_{i3T}, \quad s \in \mathcal{S} \\
& \gamma_{i,B_o} = - \left[\xi_{i1,B_o} + \beta_i(\sigma_{o0} - \sum_{j\neq i} \hat{\sigma}_{oj2} \frac{\bar{\tau}_{j0}}{\bar{c}_j}) + \gamma_i(\hat{\sigma}_{o1} - \sum_{j\neq i} \hat{\sigma}_{oj2} \frac{\bar{\tau}_{j1}}{\bar{c}_j}) \right], \\
& \gamma_{i,N_o} = - \left[\xi_{i1,N_o} + \beta_i(\mu_{o0} - \sum_{j\neq i} \hat{\mu}_{oj2} \frac{\bar{\tau}_{j0}}{\bar{c}_j}) + \gamma_i(\hat{\mu}_{o1} - \sum_{j\neq i} \hat{\mu}_{oj2} \frac{\bar{\tau}_{j1}}{\bar{c}_j}) \right],
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
d\delta_i =& -\Big\{ \frac{\alpha_i}{2}(\sigma_0^2 + \int_\Theta \mu_0^2 \nu(d\theta)) + \frac{\beta_i}{2}(\sigma_{o0}^2 + \int_\Theta \mu_{o0}^2 \nu_o(d\theta)) \\
&+ b_0\gamma_i + \gamma_{i,B_o}\sigma_{o0} + \int_\Theta \gamma_{i,B_o}(t,\theta)\mu_{o0}\nu_o(d\theta) + \sum_{s'\neq s} \tilde{q}_{ss'}[\delta_i(t,s') - \delta_i(t,s)] \\
&+ \frac{\alpha_i}{2}[\sum_{j\neq i} \hat{\sigma}_{j2}\frac{\bar{\tau}_{j0}}{\bar{c}_j}]^2 + \int_\Theta \frac{\alpha_i}{2}[\sum_{j\neq i}\hat{\mu}_{j2}\frac{\bar{\tau}_{j0}}{\bar{c}_j}]^2\nu(d\theta) + \frac{\beta_i}{2}[\sum_{j\neq i}\hat{\sigma}_{oj2}\frac{\bar{\tau}_{j0}}{\bar{c}_j}]^2 + \frac{\beta_i}{2}\int_\Theta [\sum_{j\neq i}\hat{\mu}_{oj2}\frac{\bar{\tau}_{j0}}{\bar{c}_j}]^2\nu_o(d\theta) \\
&- \frac{\bar{\tau}_{i0}^2}{2\bar{c}_i} - \alpha_i\sum_{j\neq i}\{\sigma_0\hat{\sigma}_{j2} + \int_\Theta \mu_0\hat{\mu}_{j2}\nu(d\theta)\}\frac{\bar{\tau}_{j0}}{\bar{c}_j} - \beta_i\sum_{j\neq i}\{\sigma_{o0}\hat{\sigma}_{oj2} + \int_\Theta \mu_{o0}\hat{\mu}_{oj2}\nu_o(d\theta)\}\frac{\bar{\tau}_{j0}}{\bar{c}_j} \\
&- \sum_{j\neq i}\{\gamma_{i,B_o}\hat{\sigma}_{oj2} + \int_\Theta \gamma_{i,N_o}(t,\theta)\hat{\mu}_{oj2}\nu_o(d\theta)\}\frac{\bar{\tau}_{j0}}{\bar{c}_j} - \gamma_i\sum_{j\neq i}\hat{b}_{j2}\frac{\bar{\tau}_{j0}}{\bar{c}_j} \Big\} dt \\
&- \left(\xi_{i0,B_o} + \gamma_i[\sigma_{o0} - \sum_{j\neq i}\hat{\sigma}_{oj2}\frac{\bar{\tau}_{j0}}{\bar{c}_j}]\right) dB_o - \int_\Theta \left(\xi_{i0,N_o} + \gamma_i[\mu_{o0} - \sum_{j\neq i}\hat{\mu}_{oj2}\frac{\bar{\tau}_{j0}}{\bar{c}_j}]\right)\tilde{N}_o(dt,d\theta), \\
&\delta_i(T,s) = 0,\ \delta_{i,B_o} = -\left(\xi_{i0,B_o} + \gamma_i[\sigma_{o0} - \sum_{j\neq i}\hat{\sigma}_{oj2}\frac{\bar{\tau}_{j0}}{\bar{c}_j}]\right), \\
&\delta_{i,N_o} = -\left(\xi_{i0,N_o} + \gamma_i[\mu_{o0} - \sum_{j\neq i}\int_\Theta \hat{\mu}_{oj2}\frac{\bar{\tau}_{j0}}{\bar{c}_j}]\right),
\end{aligned}
\tag{8}
$$

$$
\begin{array}{l}
A\tilde{\tau} = v,\ \bar{A}\bar{\tau}_k = \bar{v}_k,\ k \in \{0,1\},\ A_{ii} = \bar{A}_{ii} = 1, \\
j \neq i, \\
A_{ij} = \frac{\alpha_i}{c_j}[(\sigma_{i2}\sigma_{j2} + \sigma_{oi2}\sigma_{oj2}) + \int_\Theta (\mu_{i2}\mu_{j2} + \mu_{oi2}\mu_{oj2})\nu(d\theta)], \\
\bar{A}_{ij} = \frac{\alpha_i}{\bar{c}_j}\{\hat{\sigma}_{i2}\hat{\sigma}_{j2} + \int_\Theta \hat{\mu}_{i2}\hat{\mu}_{j2}\nu(d\theta)\} + \frac{\beta_i}{\bar{c}_j}\{\hat{\sigma}_{oi2}\hat{\sigma}_{oj2} + \int_\Theta \hat{\mu}_{oi2}\hat{\mu}_{oj2}\nu_o(d\theta)\}, \\
v_i = \epsilon_{i1} + \alpha_i\{b_{i2} + \sigma_1\sigma_{i2} + \int_\Theta \mu_1\mu_{i2}\nu(d\theta)\} + \alpha_i\ \{\sigma_{o1}\sigma_{oi2} + \int_\Theta \mu_{o1}\mu_{oi2}\}\nu_o(d\theta) \\
+\{\alpha_{i,B_o}\sigma_{oi2} + \int_\Theta \alpha_{i,N_o}\mu_{oi2}\nu_o(d\theta)\} + \sigma_{oi2}\alpha_i dB_o + \int_\Theta \mu_{oi2}\alpha_i\tilde{N}_o(dt,d\theta), \\
\bar{v}_{i1} = \hat{\epsilon}_{i1} + \alpha_i[\hat{\sigma}_1\hat{\sigma}_{i2} + \int_\Theta \hat{\mu}_1\hat{\mu}_{i2}\nu(d\theta)] + \beta_i[\hat{b}_{i2} + \hat{\sigma}_{o1}\hat{\sigma}_{oi2} + \int_\Theta \hat{\mu}_{o1}\hat{\mu}_{oi2}\nu_o(d\theta)] \\
+\beta_{i,B_o}\hat{\sigma}_{oi2} + \int_\Theta \beta_{i,N_o}\hat{\mu}_{oi2}\nu_o(d\theta) + \beta_i[\hat{\sigma}_{oi2}dB_o + \int_\Theta \hat{\mu}_{oi2}\ \tilde{N}_o(dt,d\theta)], \\
\bar{v}_{i0} = +\hat{\epsilon}_{i2} + \alpha_i[\sigma_0\hat{\sigma}_{i2} + \int_\Theta \mu_0\hat{\mu}_{i2}\nu(d\theta)] + \beta_i[\sigma_{o0}\hat{\sigma}_{oi2} + \int_\Theta \mu_{o0}\hat{\mu}_{oi2}\nu_o(d\theta)] \\
+\gamma_{i,B_o}\hat{\sigma}_{oi2} + \int_\Theta \gamma_{i,N_o}(t,\theta)\hat{\mu}_{oi2}\nu_o(d\theta) + \gamma_i[\hat{b}_{i2} + \hat{\sigma}_{oi2}\ dB_o + \int_\Theta \hat{\mu}_{oi2}\tilde{N}_o(dt,d\theta)], \\
c_i = r_i + \alpha_i\left(\sigma_{i2}^2 + \int_\Theta \mu_{i2}^2\nu(d\theta) + \sigma_{oi2}^2 + \int_\Theta \mu_{oi2}^2\nu_o(d\theta)\right), \\
\bar{c}_i = \hat{r}_i + \alpha_i[\hat{\sigma}_{i2}^2 + \int_\Theta \hat{\mu}_{i2}^2\nu(d\theta)] + \beta_i[\hat{\sigma}_{oi2}^2 + \int_\Theta \hat{\mu}_{oi2}^2\nu_o(d\theta)],
\end{array}
\tag{9}
$$

bDIPONET: blockchain-based distributed power networks



Ingredients of the interaction

- Platform: blockchain-based distributed power network
- Players: producers, consumers, prosumers, Wattcoin miners
- Action: price, quantity
- State: reserve, remaining energy/budget, price
- Payoff: revenue, bill-saving, demand-supply mismatch
- Mean-field effect: peak hours, total demand per area, variance

What is a Blockchain?

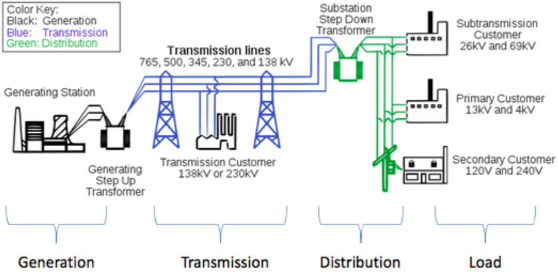
- Distributed ledger, records transactions and ownership, operated within a peer to peer network
- Wattcoin Blockchain: ownership of Wattcoins (energy coins)

image credit: coinmarket

What is Wattcoin Mining?

Decentralized process:

- Feasibility: Confirm the feasibility of energy service from the generation to the end user.
- users' Quality-of-Experience (QoE)-based mining
- Valid: Create new block if trustful QoE- transaction



How to check the validity of demand-supply?

- New feasible energy transactions are broadcast to all nodes.
- Each node collects new transactions into a block.
- Each node works on finding a proof-of-validity for its block. When a node finds a proof-of-validity, it broadcasts the block to all nodes.
- Nodes accept the block only if all transactions in it are valid and not already spent.
- Nodes express their acceptance of the block by working on creating the next block in the chain, using the hash of the accepted block as the previous hash.

To which previously solved block will miners chain their block?

Consensus $\Longrightarrow$ Longest chain rule (LCR)
Nakamoto (2008): Nodes consider the longest chain to be the correct one and will keep working on extending it.
If miners follow LCR : no deviation, there is a consensus !

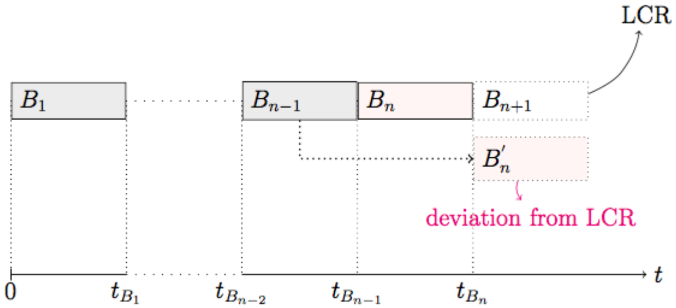
On March 13, 2013 (coinmagazine.com) : "Starting from block 225430, the blockchain literally split into two, with one half of the network adding blocks to one version of the chain, and the other half adding to the other. [...] The split lasted for 24 blocks or 6 hours ..."

Wattcoin mining

- all miners observe blocks and transactions simultaneously
- Strategies: to which block do they chain their own block
- Payoffs: When miner solves block B_n in Wattcoin blockchain, he broadcasts this, including his reward, in Wattcoins, in block B_n
- However this reward can't be spent immediately, only after sufficiently many blocks chained to B_n
- Goal: Miners want to mine on chain which they think will become consensus, so their rewards are valuable.

This is dynamic **crowd-seeking** game !

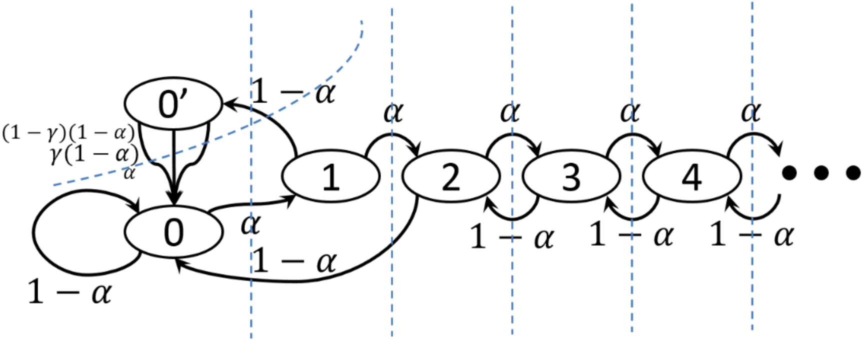
Deviation from the Longest chain



Co-opetition rather than competition

- Miners are not really competing to solve their block before the others
- That someone else solves his block before me, does not, in itself, reduce my gains
- The only thing that matters is that we all cooperate to coordinate well, and all work on the same chain, so that we all obtain maximum rewards for the blocks we solve

State dynamics



Risk-aware co-opetition between Wattcoin miners

- Public branches $:= B$
- m_b =fraction of miners in the public branch b
- Crowd effect: $\alpha = \alpha_0 + \epsilon P(x(t) = k | u =$ same public branch$) m_b$
- reward: $r_j \bar{p} x(T)$, instant mining cost $c(t, .)$
- $R_j(u) = r_j \bar{p} x(T) - \int_0^T c(t, .) dt$
- Payoff: $\frac{1}{\eta_j} \log \left[E e^{\eta_j R_j(u)} \right]$

Motivation/Review

Models of competition among producers have been discussed in simple terms

Motivation

In the context of Smart Grids

Models of competition among producers have been discussed in simple terms

Motivation

Models of competition among producers have been discussed in simple terms:

1838	1922	1925	1978	1987	2000
Cournot	Evans	Roos	Simaan & Takayama	Fershtman & Kamien	Docker et. al.

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Models of competition among producers have been discussed in simple terms:

- 1838 Cournot
- 1922 Evans
- 1925 Roos
- 1978 Simaan & Takayama
- 1987 Fershtman & Kamien
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Cournot → **Cournot Model**

Augustin Cournot, Recherches sur les principes mathematiques de la theorie des richesses, Paris, 1838; translated by N. T. Bacon, Researches into the Mathematical Principles of the Theory of Wealth, London, 1897; see chapter VII.

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Cournot Model for quadratic production cost output

Griffith C. Evans, A Simple Theory of Competition, The American Mathematical Monthly, Vol. 29, No. 10 (Nov. - Dec., 1922), pp. 371- 380

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[Page 163, Roos 1925] Defines an open-loop solution concept for deterministic differential Cournot games (open-loop NE)

C.F. Roos, A Mathematical Theory of Competition, American Journal of Mathematics, 47, 163-175, 1925

C.F. Roos, A Dynamic Theory of Economics, Journal of Political Economy 35, 632-656, 1927

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Considering capacity constraints

M. Simaan and T. Takayama, Game Theory Applied to Dynamic Duopoly with Production Constraints, Automatica (14) 161-166, 1978

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Open-loop and closed-loop NE for a class of deterministic differential Cournot games

C. Fershtman and M. I. Kamien, Dynamic Duopolistic Competition with Sticky Prices Econometrica, Vol. 55, No. 5, pp. 1151-1164, 1987

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2000 Docker et. al.

Dynamic Cournot oligopoly problems

E. J. Dockner, S. Jorgensen, N. Van Long, and G. Sorger, Differential Games in Economics and Management Science, Cambridge University Press, Nov 16, - Business and Economics, 2000

Producers

here: Stochastic dynamic Cournot game among electricity producers

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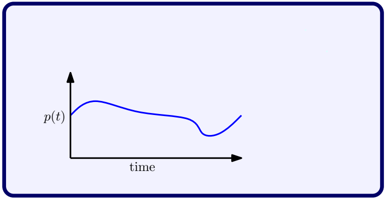
Price of electricity $p(t)$



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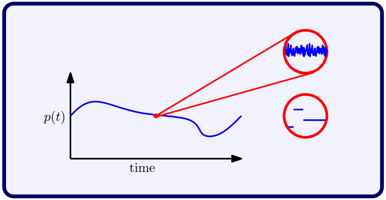
Producers

here: Stochastic dynamic Cournot game among electricity producers

Price of electricity $p(t)$ is adjusted progressively with: $\begin{cases} \text{(I)} & \text{Local uncertainty} \\ \text{(II)} & \text{Global uncertainty} \end{cases}$



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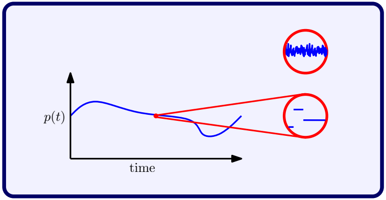
Price of electricity $p(t)$ is adjusted progressively with:

- (I) Local uncertainty
- (II) Global uncertainty
- (III) Jump terms



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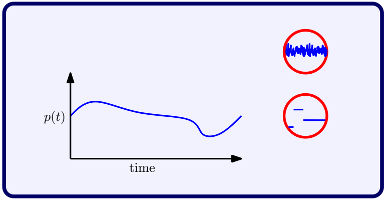

Our model:
Similar to [Roos]
+ Risk-Minimization
terms [Markowitz]

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Key results

- We formulate a mean-field-type game with common noise and jump-diffusion
- We provide semi-explicit solution
- Mean-field term $\Rightarrow$
  - Conditional expectation of the price
  - w.r.t the filtration generated by the common noise
- Mean-field interaction is stochastic

Production under estimated demand

$\mathcal{T} := [t_0, t_1]$ time horizon

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Log-price dynamics with $p(t_0) = p_0$

$$dp = \eta[a - D - p]dt + \left(\sigma \, dB + \int_{\theta \in \Theta} \mu(\theta) \, \tilde{N}(dt, d\theta)\right) + \sigma_o \, dB_o + \int_{\theta \in \Theta} \mu_o(\theta) \tilde{N}_o(dt, d\theta)$$

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supply $D(t) := \sum_{i=1}^{n} u_i(t)$

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dB: standard Brownian motion

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standard Brownian motion (dB)

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jump space (Θ)

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jump rate size in $L^2_\nu(\Theta, \mathbb{R})$
$\int_{\theta\in\Theta} \mu^2(\theta)\nu d\theta < +\infty$

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$\mathcal{F}_t^{B_o}$	filtration generated by the observed common noise up to $t \in \mathcal{T}$

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- dB, dB_o: standard Brownian motion
- supply $D(t) := \sum_{i=1}^{n} u_i(t)$
- Θ: jump space
- $\mu(\theta)$: jump rate size in $L^2_\nu(\Theta, \mathbb{R})$: $\int_{\theta \in \Theta} \mu^2(\theta)\nu d\theta < +\infty$

$\mathcal{F}_t^{B_o}$	filtration generated by the observed common noise up to $t \in \mathcal{T}$
$\eta = \tilde{s}m_b > 0$	adjustment of market gain
a, σ, σ_0	fixed parameters

Production under estimated demand

At $t \in \mathcal{T}$, producer i receives

$$\hat{p}(t)u_i - C_i(u_i),$$

$\hat{p}(t) = \mathbb{E}\left(p|\mathcal{F}_t^{B_o}\right)$ Conditional expectation of the price given B_o

$\hat{u}_i(t) = \mathbb{E}\left(u_i|\mathcal{F}_t^{B_o}\right)$ Conditional expectation of producer i's output given B_o

$c_i = \tilde{c}_i + \epsilon D_i + \hat{c}_{i,miners}$

$$C_i(u_i) = c_i u_i + \frac{1}{2} r_i u_i^2 + \frac{1}{2}\bar{r}_i \hat{u}_i^2$$

Captures the risk sensitivity of producer i

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Long-term revenue of producer i:

$$\mathcal{R}_{i,\mathcal{T}}(p_0, u) = -\frac{q}{2}e^{-\lambda_i t_1}\left(p(t_1) - \hat{p}(t_1)\right)^2 + \int_{t_0}^{t_1} e^{-\lambda_i t}\left[\hat{p}u_i - C_i(u_i)\right]dt$$

Why is this a mean-field-type game?

- Strategic game: coupled producers through price functional

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- Two conditional mean-field terms:
  - Conditional price $\hat{p}(t)$ based on observations of B_o, N_o
  - Square of the conditional control input $\hat{u}^2(t) = \left(\mathbb{E}\left[u_i | \mathcal{F}_t^{p_0, B_o, N_o}\right]\right)^2$ based on observations of B_o, N_o

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Best Response BR_i

Any strategy $u_i^*(\cdot) \in \mathcal{U}_i$ satisfying the maximum in

$$\begin{cases} \sup_{u_i \in \mathcal{U}_i} \mathbb{E}\left[R_{i,\mathcal{T}}(p_0, u)\right], \\ dp = \eta\left[a - D - p\right] dt + \left(\sigma dB + \int_{\Theta} \mu(\theta)\tilde{N}(dt, d\theta)\right) + \sigma_o dB_o + \int_{\theta\in\Theta} \mu_o(\theta)\tilde{N}_o(dt, d\theta), \\ p(t_0) = p_0, \end{cases} \tag{10}$$

is called a best-response strategy of producer i to the other producers strategy $u_{-i} \in \prod_{j\neq i} \mathcal{U}_j$.

Finding state-and-mean-field feedback Nash solution

Open-loop equilibrium in (deterministic) differential games [Roos,1925]

C.F. Roos, A Mathematical Theory of Competition, American Journal of Mathematics, 47, 163-175, 1925

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Open-loop equilibrium in (deterministic) differential games [Roos,1925]

In the open-loop setting:

Producer information structure: $\begin{cases} \text{Common noise } B_o, N_o \\ \text{Production output function of } t \text{ and } \mathcal{F}^{p_0, B_o, N_o} - \text{measurable} \end{cases}$

Control action law $\phi_i(t, p_0, B_o, N_o)$

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State-and-mean-field feedback:

Producer information structure:

$\begin{cases} \text{Common noise } B_o, N_o \\ \text{Production output function of } t \text{ and } \mathcal{F}^{p_0, p, B_o, N_o} - \text{measurable} \end{cases}$

Control action law $\phi_i(t, p_0, p, \hat{p}, B_o, N_o)$

C.F. Roos, A Mathematical Theory of Competition, American Journal of Mathematics, 47, 163-175, 1925

Finding state-and-mean-field feedback Nash solution

Open-loop equilibrium in (deterministic) differential games [Roos,1925]

In the open-loop setting:

Producer information structure: $\begin{cases} \text{Common noise } B_o, N_o \\ \text{Production output function of } t \text{ and } \mathcal{F}^{p_0, B_o, N_o} - \text{measurable} \end{cases}$

Control action law $\phi_i(t, p_0, B_o, N_o)$

State-and-mean-field feedback:

Producer information structure:

$\begin{cases} \text{Common noise } B_o, N_o \\ \text{Production output function of } t \text{ and } \mathcal{F}^{p_0, p, B_o, N_o} - \text{measurable} \end{cases}$

Control action law $\phi_i(t, p_0, p, \hat{p}, B_o, N_o)$

A state-and-mean-field feedback Nash equilibrium is a strategy profile $u_i \in \mathcal{U}_i^{fb}$ such that $u_i \in BR_i((u_j)_{j\neq i})$ and $u_i(t)$ can be expressed as function of $(t, p(t), \hat{p}(t))$

C.F. Roos, A Mathematical Theory of Competition, American Journal of Mathematics, 47, 163-175, 1925

Mean-field-type equilibria

For each i, the equilibrium strategy is in state-and-conditional mean-field feedback form:

$$u_i^* = -\frac{\eta\tilde{\alpha}_i}{r_i}(p-\hat{p}) + \frac{\hat{p}(1-\eta\tilde{\beta}_i) - (c_i+\eta\tilde{\gamma}_i)}{r_i+\bar{r}_i},$$

where the conditional equilibrium price:

$$\begin{cases} d\hat{p} = \eta\left\{a + \sum_{j=1}^{n}\frac{c_j+\eta\tilde{\gamma}_j}{r_j+\bar{r}_j} - \hat{p}\left(1+\sum_{j=1}^{n}\frac{1-\eta\tilde{\beta}_j}{r_j+\bar{r}_j}\right)\right\}dt + \sigma_o dB_o + \int_\Theta \mu_o \tilde{N}_o(dt,d\theta), \\ \hat{p}(t_0) = \hat{p}_0, \end{cases}$$

and the stochastic parameters $\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}, \tilde{\delta}$ solve the stochastic Riccati system:

$$\begin{cases} d\tilde{\alpha}_i = \left\{(\lambda_i+2\eta)\tilde{\alpha}_i - \frac{\eta^2}{r_i}\tilde{\alpha}_i^2 - 2\eta^2\tilde{\alpha}_i\sum_{j\neq i}\frac{\tilde{\alpha}_j}{r_j}\right\}dt + \tilde{\alpha}_{i,o}dB_o + \int_\Theta \tilde{\mu}_{io}\tilde{N}_o(dt,d\theta), \\ \tilde{\alpha}_i(t_1) = -q, \\ d\tilde{\beta}_i = \left\{(\lambda_i+2\eta)\tilde{\beta}_i - \frac{(1-\eta\tilde{\beta}_i)^2}{r_i+\bar{r}_i} + 2\eta\tilde{\beta}_i\sum_{j\neq i}\frac{1-\eta\tilde{\beta}_j}{r_j+\bar{r}_j}\right\}dt + \tilde{\beta}_{i,o}dB_o + \int_\Theta \tilde{\mu}_{io}\tilde{N}_o(dt,d\theta), \\ \tilde{\beta}_i(t_1) = 0, \end{cases}$$

Mean-field-type equilibria

$$
\begin{cases}
d\tilde{\gamma}_i = \Big\{(\lambda_i+\eta)\tilde{\gamma}_i - \eta\tilde{\beta}_i a - \tilde{\beta}_{i,o}\sigma_o + \frac{(1-\eta\tilde{\beta}_i)(c_i+\eta\tilde{\gamma}_i)}{r_i+\bar{r}_i} + \eta\tilde{\gamma}_i\sum_{j\neq i}\frac{1-\eta\tilde{\beta}_j}{r_j+\bar{r}_j} \\
\qquad -\eta\tilde{\beta}_i\sum_{j\neq i}\frac{c_j+\eta\tilde{\gamma}_j}{r_j+\bar{r}_j}\Big\}dt - \tilde{\beta}_i\sigma_o dB_o + \int_\Theta \mu_{o,\beta}\tilde{N}_o(dt,d\theta), \\
\tilde{\gamma}_i(0) = 0, \\
d\tilde{\delta}_i = -\Big\{-\lambda_i\tilde{\delta}_i + \frac{1}{2}\sigma_o^2\tilde{\beta}_i + \frac{1}{2}\tilde{\alpha}_i\left(\sigma^2 + \int_\Theta \mu^2(\theta)\nu(d\theta)\right) + \eta\tilde{\gamma}_i a \\
\qquad +\tilde{\gamma}_{i,o}\sigma_o + \frac{1}{2}\frac{(c_i+\eta\tilde{\gamma}_i)^2}{r_i+\bar{r}_i} + \eta\tilde{\gamma}_i\sum_{j\neq i}\frac{c_j+\eta\tilde{\gamma}_j}{r_j+\bar{r}_j}\Big\}dt - \sigma_o\tilde{\gamma}_i dB_o + \int_\Theta \tilde{\mu}_{io}\tilde{N}_o(dt,d\theta), \\
\tilde{\delta}_i(t_1) = 0.
\end{cases}
$$

The equilibrium revenue of producer i is

$$
\mathbb{E}\frac{1}{2}\alpha_i(t_0)(p(t_0)-\hat{p}_0)^2 + \frac{1}{2}\beta_i(t_0)\hat{p}_0^2 + \gamma_i(t_0)\hat{p}_0 + \delta_i(t_0).
$$

Interior static vs stationary feedback equilibrium price

Stationary feedback equilibrium price

$$\bar{p}^* = \frac{a + \sum_{j=1}^{n} \frac{c_j + \eta\tilde{\gamma}_j}{r_j + \bar{r}_j}}{1 + \sum_{j=1}^{n} \frac{1 - \eta\tilde{\beta}_j}{r_j + \bar{r}_j}},$$

One-shot Cournot equilibrium price

$$p^C = \frac{a + \sum_{j=1}^{n} \frac{c_j}{1 + r_j + \bar{r}_j}}{1 + \sum_{j=1}^{n} \frac{1}{1 + r_j + \bar{r}_j}}$$

Simulation Results: three different scenarios

1. Deterministic scenario without control, i.e., $\sigma = \sigma_0 = 0$ and

Table: Parameters Scenario 1

Parameter	value
t_1	1
p_0	170
s	100
a	30
D	$30 + 20\sin(10\pi t)$

Simulation Results: three different scenarios

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Table: Parameters Scenario 1

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t_1	1
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D	$30 + 20\sin(10\pi t)$

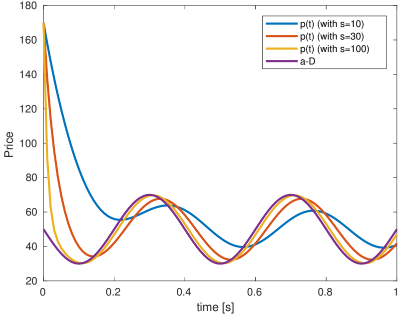


Figure: Evolution of the price given some demand profile for different values of s. This shows a tracking behavior.

Simulation Results: three different scenarios

1. Deterministic scenario without control, i.e., $\sigma = \sigma_o = 0$
2. Deterministic scenario with control, i.e., $\sigma = \sigma_o = 0$ and

Table: Parameters Scenarios 2

Parameter	**value**
t_1	1.5
p_0	50
$\hat{p}_0$	50
c_1	1
c_2	5
c_3	10
$\lambda_1, \ldots, \lambda_3$	0.1
η	0.5
a	1
$r_1, \bar{r}_1$	1
$r_2, \bar{r}_2$	2
$r_3, \bar{r}_3$	3
q	1
D	$u_1 + u_2 + u_3$

Results

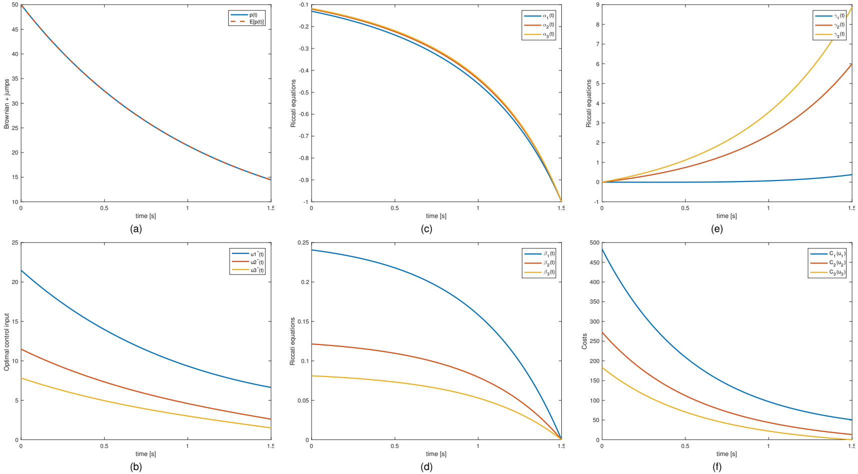


Figure: Mean-field-type game performance corresponding to Scenario 2. Figures show: (a) $p(t)$ and $\mathbb{E}[p(t)]$, (b) $u_i^*(t)$, (c)-(e) Riccati equations $\alpha_i(t), \beta_i(t), \gamma_i(t)$, and (f) costs $C_i(u_i)$, for all $i = \{1, \ldots, 3\}$.

Simulation Results: three different scenarios

1. Deterministic scenario without control, i.e., $\sigma = \sigma_o = 0$
2. Deterministic scenario with control, i.e., $\sigma = \sigma_o = 0$
3. Stochastic scenario with jump terms and control, i.e., $\sigma = 1$, and

Table: Parameters Scenarios 2

Parameter	value
t_1	1.5
p_0	50
$\hat{p}_0$	50
c_1	1
c_2	5
c_3	10
$\lambda_1, \ldots, \lambda_3$	0.1
η	0.5
a	1
$r_1, \bar{r}_1$	1
$r_2, \bar{r}_2$	2
$r_3, \bar{r}_3$	3
q	1
D	$u_1 + u_2 + u_3$

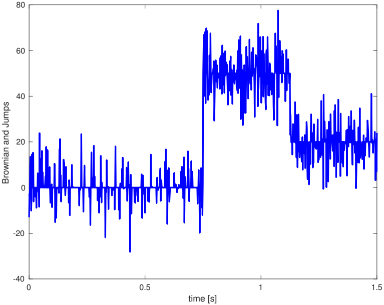


Figure: Brownian motion + two jumps.

Results

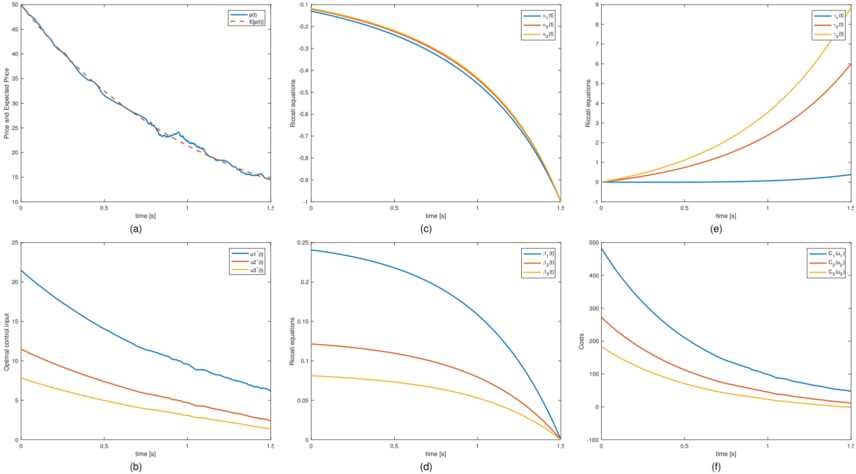
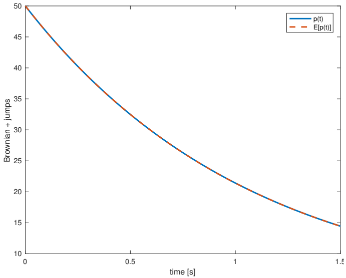


Figure: Mean-field-type game performance corresponding to Scenario 3. Figures show: (a) $p(t)$ and $\mathbb{E}[p(t)]$, (b) $u_i^*(t)$, (c)-(e) Riccati equations $\alpha_i(t)$, $\beta_i(t)$, $\gamma_i(t)$, and (f) costs $C_i(u_i)$, for all $i \in \{1, \ldots, 3\}$.

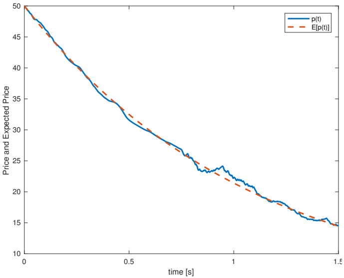
Costs comparison

Table: Obtained costs for Scenarios 1 and 2

Scenario	C_1	C_2	C_3
2	180430	93526	57159
3	180570	93608	57224



(a)



(b)

Figure: Mean-field-type game performance corresponding to (a) Scenario 2 and (b) Scenario 3. Price evolution $p(t)$ and $\mathbb{E}[p(t)]$.

summary I: given a demand profile

- We have examined a price formation in smart energy systems using a price dynamics model introduced by Roos in 1925
- We have introduced a common noise in the environment
- Producers can condition on B_o and exploit that information
- The new conditioned price affects the revenues of the producers
- We have provided explicit solution to the master system of the corresponding mean-field-type game with jump-diffusion and common noise
- It is shown that the optimal strategies are in state-and-mean-field feedback form.

Generation-transmission-distribution

- nodes: generators+frequency-response devices+passive buses
- each node with a voltage waveform $V_i \cos(\omega t + \theta_i),\ \omega = 2\pi f,\ f = 50Hz.$

$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i + u_i - \sum_{j\in\mathcal{V}} B_{ij}\sin(\theta_i - \theta_j), \forall i \in \mathcal{V}_g,$$

$$D_i\dot{\theta}_i = P_i + u_i - \sum_{j\in\mathcal{V}} B_{ij}\sin(\theta_i - \theta_j), \forall i \in \mathcal{V}_f,$$

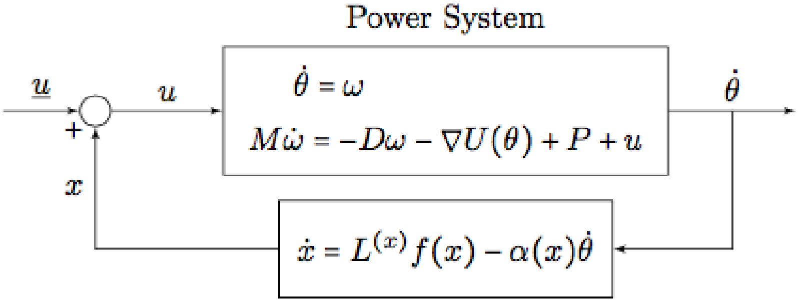
$$0 = P_i + u_i - \sum_{j\in\mathcal{V}} B_{ij}\sin(\theta_i - \theta_j), \forall i \in \mathcal{V}_p,$$

J. Barreiro-Gomez, F. Dörfler, and H. Tembine. Distributed and Robust Population Games with Applications to Optimal Frequency Control in Power Systems. In American Control Conference, Milwaukee, WI, July 2018.

Closed-loop frequency control

$$\min_{u \in \mathbb{R}^b} \sum_{i \in \mathcal{V}} J_i(u_i), \text{ s.t.}$$

$$0 = \sum_{i \in \mathcal{V}} P_i + u_i, \ \ \underline{u}_i \le u_i \le \bar{u}_i, \ \forall i \in \mathcal{V},$$



Producers: how to match the demand?

$$L_j(s_j, e(0)) = l_{jT}(e(T), var(e(T))) + \int_0^T l_j(D_j(t) - S_j(t))\, dt$$

Demand-Supply Matching

$$\begin{cases} \text{mismatch cost } \inf_{s_j} \mathbb{E} e^{\eta_j L_j(s_j, e(0))} \\ de_{jk}(t) = [a_{jk}(t) - s_{jk}(t)]dt + \sigma dB + \int \mu(t,\theta)\tilde{N}(dt, d\theta) \\ \text{Arrival rate } a_{jk}(t) \geq 0. \\ \text{Quantity: } s_{jk}(t) \in [0, \bar{s}_{jk}],\ j, k \end{cases}$$

Structure of the solution

$PS_n =$ n-th cheapest power stations

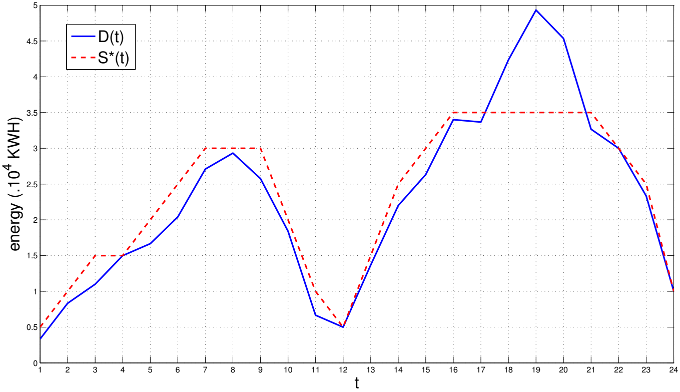
maximum production capacity: $W_n = \sum_{jk \in PS_n} \bar{s}_{jk} \mathbb{1}_{k \in A_j}$

$\underline{p}_n =$ cheapest cost $\min_{jk \notin (\cup_{k=1}^{n-1} PS_k)} p_{jk}$

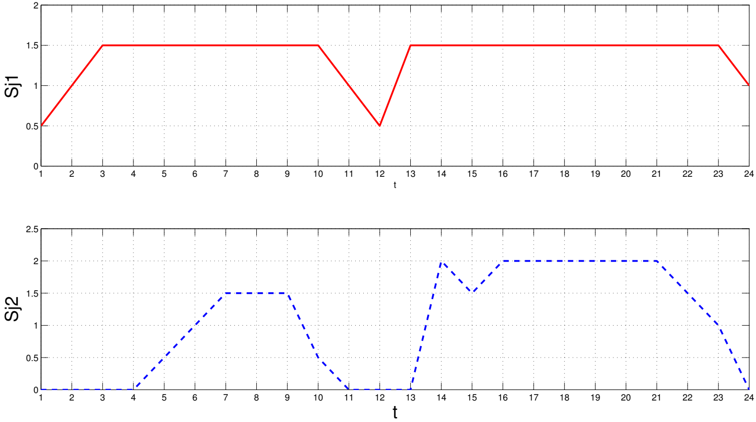
Optimal control is not unique

$$S_j^* = D_j - (l_j')^{-1}(\underline{p}_n) \; if \; D_j(t) \in (\sum_{k=1}^{n-1} W_k, \sum_{k=1}^{n} W_k)$$

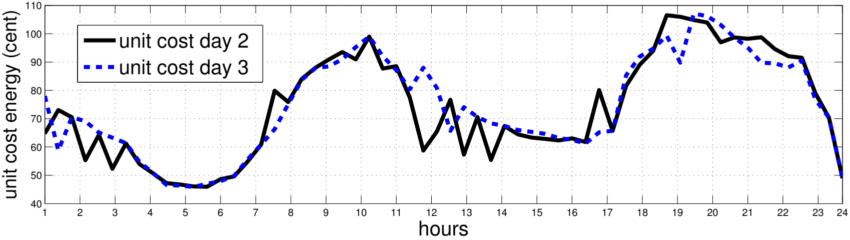
Optimal production strategies

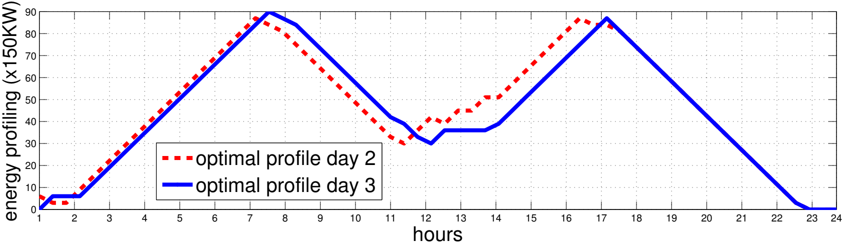


Supplied by two power plants



Price Dynamics





Consumer response

Consumer payoff

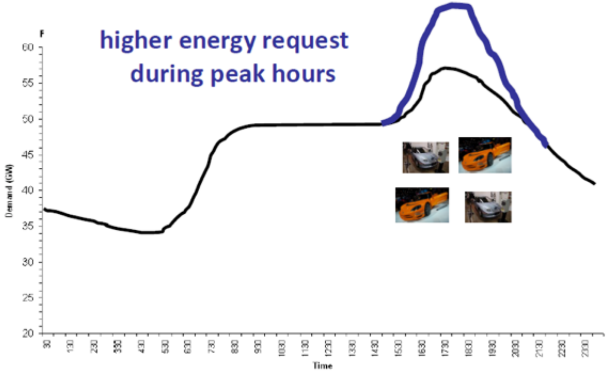
$$r_i = w_i(d_i) - p(D, S)d_i.$$

Optimal demand: nonatomic case

$$d_i^* = \max(0, (w_i')^{-1}[p]).$$

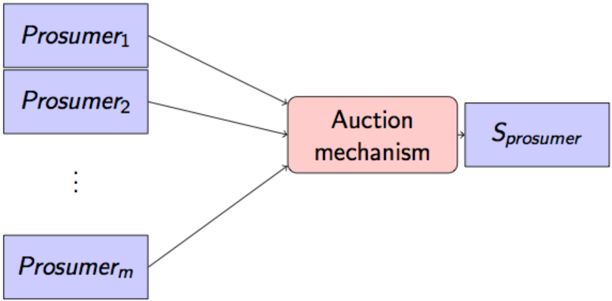
The optimal demand depends on the price and the price depends on the supply.

Prosumers market: Demand-Supply mismatch



Blockchain-based prosumers' market to cover Demand-Supply mismatch ?

Prosumers market auction



Prosumers market interaction: auction

Payoff

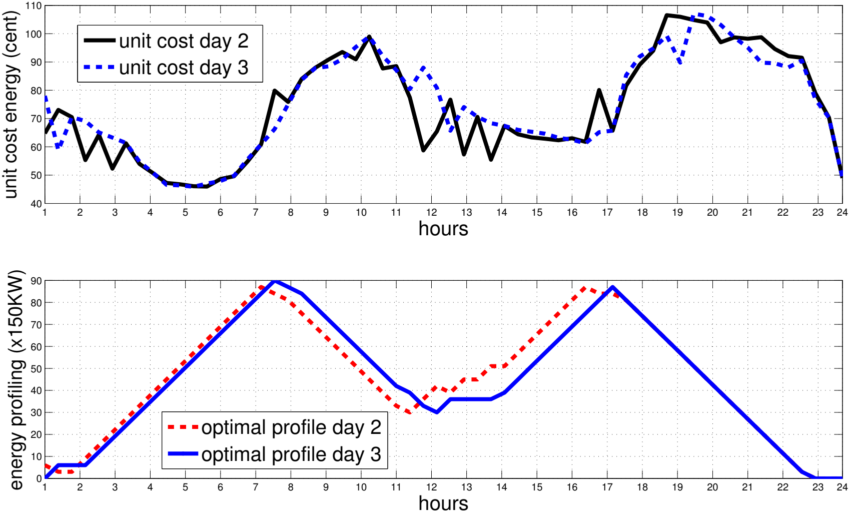
- $(p_i - c_i) q_i \mathbb{1}_{i \in winners}$
- $G_i(y) = P(c_i > y)$.
- Prosumer i knows the c_i but not $(c_j)_{\neq i}$

Optimal bidding strategy

$$\begin{cases} \lambda_i'(x) = \frac{G_i(\lambda_i(x))}{G_i'(\lambda_i(x))} \left[\frac{1}{x - \lambda_i(x)} - \frac{1}{K-1} \sum_{k=1}^{K} \frac{1}{x - \lambda_k(x)} \right] \\ \lambda_i(\min_k p_k) = \min_k \lambda_k, \ \lambda_i(\max_k p_k) = \max_k \lambda_k. \end{cases}$$

The optimal bidding price increases with the supply cost

Price Dynamics



Prosumers market interaction: consumption

Payoff i

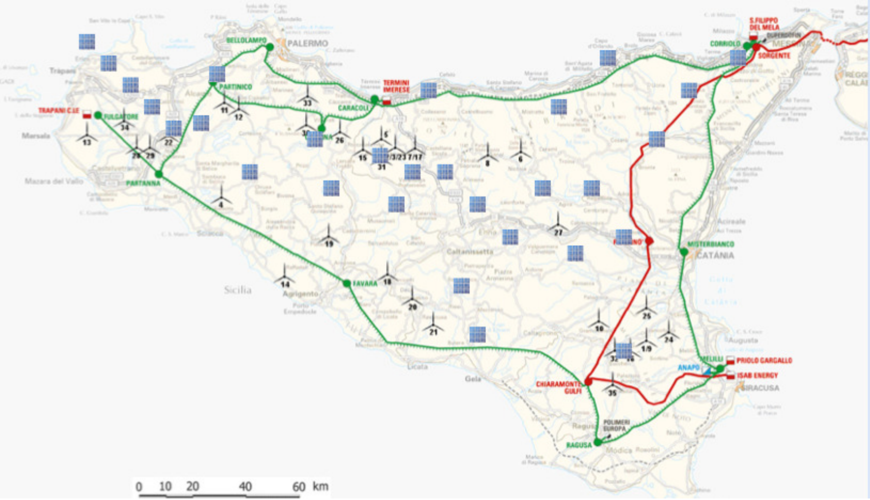
- $\hat{r}_i := -c_i(s_i) + [w_i(-\hat{q}_i) + p(D,S)\hat{q}_i]\, \mathbb{1}_{\{\hat{q}_i<0\}} + [w_i(d_i) + p_i q_i]\, \mathbb{1}_{\{\hat{q}_i>0\}} \mathbb{1}_{\{i \in Winners)\}}$

Optimal strategy of the prosumer

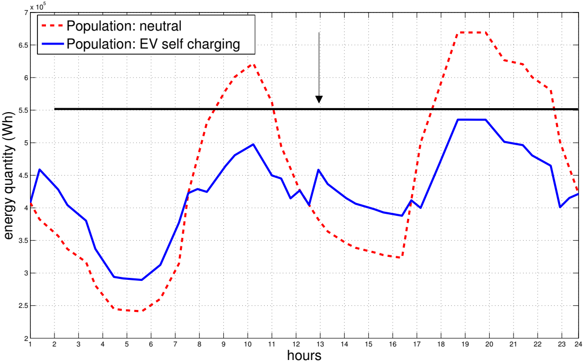
$$\begin{cases} c_i'(x+d_i+\epsilon) - w_i'(-x) + xp_D + p = 0 & \text{if } \hat{q}_i(t) < 0 \\ (\hat{p}_i(t), \hat{q}_i(t)) & \text{if } \hat{q}_i(t) > 0 \end{cases}$$

Prosumer strategy based on the available energy reserve

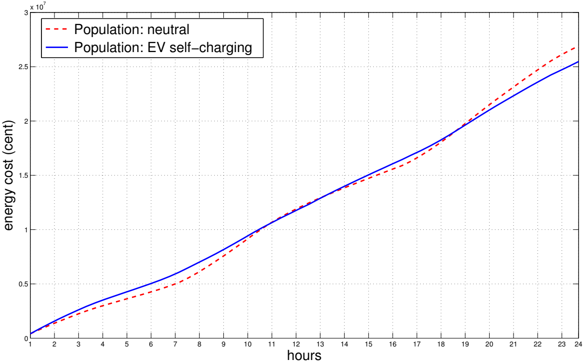
Reducing NegaWatts? Sicily region (Italy)



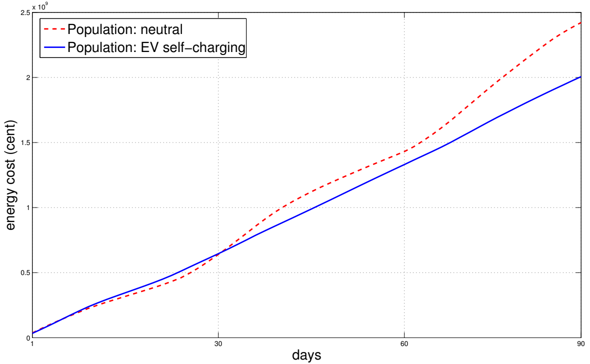
EV penetration: saving by 17,91 %.



EV penetration: total energy cost saving about 6%.



EV penetration 3 months duration: saving about 19%.



Wattcoin for Underserved Areas?

- Why would a prosumer adopt a blockchain platform?
  - incentive : Wattcoin
  - know your users' need
- b-DIPONET
  - reduces distribution cost (generation is near the point-of-use)
  - improves quality-of-experience (to the end user)
  - reduces outage

Blockchain-based technology: example of unstable price

Do we want energy price to be like the bitcoin price 2017-2018 ?

Figure: Coindesk database: the price of bitcoin went from 10K USD to 20 K USD and back to below 7 K USD within 2-3 months in 2017-2018.

Blockchain-based technology: example of a more stable price

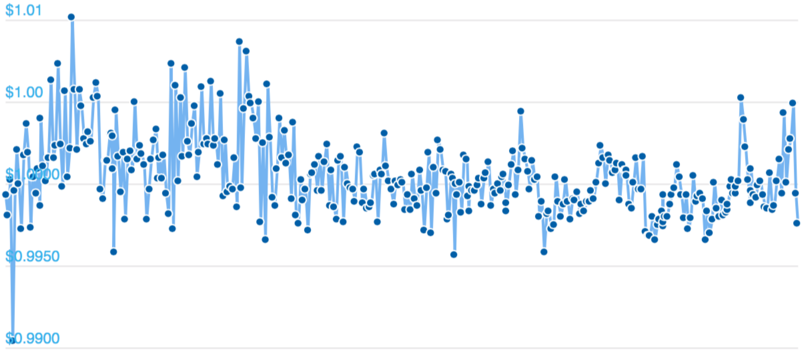


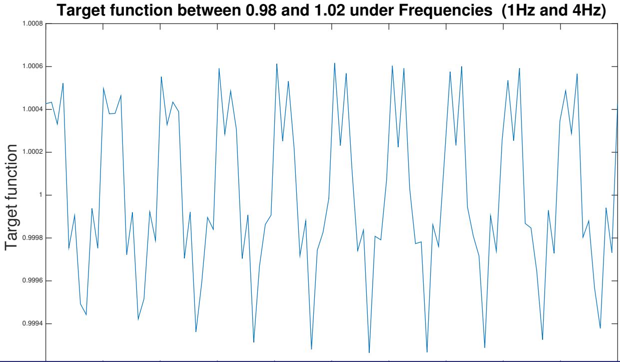
Figure: Coindesk database: the price of tether USD went from 0.99 USD to 1.01 USD

Construction of a regulated price

choose the coefficients $c, \hat{c}$ such that target price $p_{tp,i}(t) \in [\underline{p}_i, \bar{p}_i]$

$$p_{tp,i}(t) = c_{i0} + \sum_{k=1}^{2} c_{ik} \cos(2\pi k t) + \hat{c}_{ik} \sin(2\pi k t),$$

Example: $c_{i0} := \frac{\underline{p}_i + \bar{p}_i}{2}, c_{i1} := \frac{\bar{p}_i - \underline{p}_i}{100}, \hat{c}_{i1} := \frac{\bar{p}_i - \underline{p}_i}{150}, c_{12} := \frac{\bar{p}_i - \underline{p}_i}{200}, \hat{c}_{12} := \frac{\bar{p}_i - \underline{p}_i}{250}.$



A more stable and regulated price

$$\left\{\begin{array}{l} \text{Asset state in Wattcoin:} \\ dp_i = \eta_i[D_i - p_i - (S_i + u_i)]dt + \left(\sigma_i dB_i + \int_{\theta\in\Theta} \mu_i(\theta)\tilde{N}_i(dt, d\theta)\right) \\ + \left(\sigma_o dB_o + \int_{\theta\in\Theta} \mu_o(\theta)\tilde{N}_o(dt, d\theta)\right), \\ \\ \text{Asset cost functional:} \\ L_{mftg} = q_i(t_1)var(p_i(t_1) - p_{tp,i}(t_1)) + [q_i(t_1) + \bar{q}_i(t_1)][Ep_i(t_1) - p_{tp,i}(t_1)]^2 \\ + \int_{t_0}^{t_1} q_i(t)var(p_i(t) - p_{tp,i}(t)) + (q_i(t) + \bar{q}_i(t))[Ep_i(t) - p_{tp,i}(t)]^2 \, dt. \end{array}\right. \quad (11)$$

A more stable and self-regulated energy price

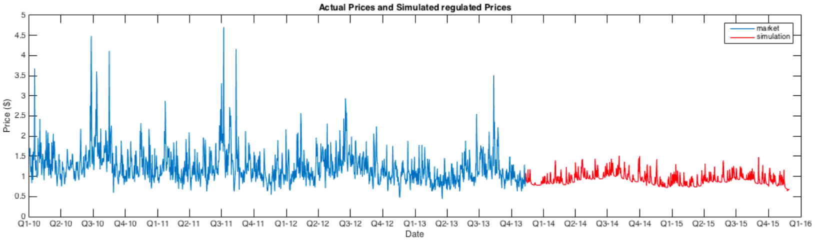


Figure: Real energy market price and simulation of the regulated price dynamics as a continuation price under MFTG strategy.

Consumption-Investment-Insurance over several energy assets

Some prior works

Mossin, J. (1968), Aspects of rational insurance purchasing, Journal of Political Economy, 79: 553-568.

van Heerwaarden, A. (1991), Ordering of Risks, Thesis, Tinbergen Institute, Amsterdam.

Kristen S. Moore and Virginia R. Young, Optimal Insurance in a Continuous-Time Model, Insurance: Mathematics and Economics 39, 47-68, 2006.

MFTG: Consumption-Investment-Insurance

$$
\begin{array}{l}
\sup_{(u^c,u^I,u^{ins})} \; -qe^{-\lambda t_1}[s(t_1)-\hat{s}(t_1)]^2 + \int_{t_0}^{t_1} e^{-\lambda t} \log u^c \, dt \\
ds = \kappa_0(r_0(\tilde{s})+\hat{\mu}_0(\tilde{s}))sdt \\
+\sum_{k=1}^{d}[\hat{\mu}_k - (r_0(\tilde{s})+\hat{\mu}_0(\tilde{s}))\kappa_0 + \text{Drift}_k(\tilde{s})]u_k^I dt \\
-u^c dt - \bar{\lambda}(\tilde{s})(1+\bar{\theta}(\tilde{s}))E[u^{ins}]dt \\
+\sum_{k=1}^{d} u_k^I \text{Diffusion}_k(\tilde{s}) + \sum_{k=1}^{d} u_k^I \text{Jump}_k(\tilde{s}), \\
-(L-u^{ins})dN,
\end{array}
\tag{12}
$$

$$
\begin{array}{l}
L = l(\tilde{s})s, \\
Drift_k = \eta_k[D_k - p_k - (S_k + u_{mftg,k})] + \frac{1}{2}(\sigma_k^2+\sigma_o^2) \\
+\int_{\Theta}[e^{\mu_k}-1-\mu_k]\nu(d\theta) + \int_{\Theta}[e^{\mu_o}-1-\mu_o]\nu_o(d\theta), \\
Diffusion_k = (\sigma_k dB_k + \sigma_o dB_o), \\
Jump_k = \int_{\Theta}[e^{\mu_k}-1]\tilde{N}_k(dt,d\theta) + \int_{\Theta}[e^{\mu_o}-1]\tilde{N}_o(dt,d\theta),
\end{array}
\tag{13}
$$

MFTG: Consumption-Investment-Insurance

Investment over several energy asset

MFTG strategy

Consumption

the optimal consumption strategy process is proportional to the wealth process: $u^c = \frac{e^{-\lambda t}}{\alpha_1} s$

Insurance

the optimal insurance is a decreasing and convex function of $\bar{\theta}$.

$$u^{ins} := \left[l(\tilde{s}) - \frac{1+\theta(\tilde{s})}{2+\theta(\tilde{s})} \right]_+ s.$$

Remarks

Blockchain-based distributed power networks

1. Identified blockchain-based interactions
2. Opportunities:
   - Reduced peak,
   - Improved penetration of renewables by means of incentives,
   - Improved adoption of alternatives energy by means of Wattcoin,
   - Improved users' quality-of-experience service in under-served areas

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Thank you very much for your attention!