Game Theoretic Models for Energy Production

Ronnie Sircar

Department of Operations Research and Financial Engineering
Bendheim Center for Finance
Program in Applied & Computational Mathematics
Andlinger Center for Energy & the Environment

Princeton University
http://www.princeton.edu/∼sircar

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- Recent steep decline in oil prices (around $110 per barrel in June 2014 to ∼ $30 in April 2016, currently ∼ $70) :

- Drop was prompted in large part by OPEC's strategic decision not to decrease its oil output in the face of increased production of *shale oil* in the US, coming from fracking.

Heterogeneous Costs: just oil sources

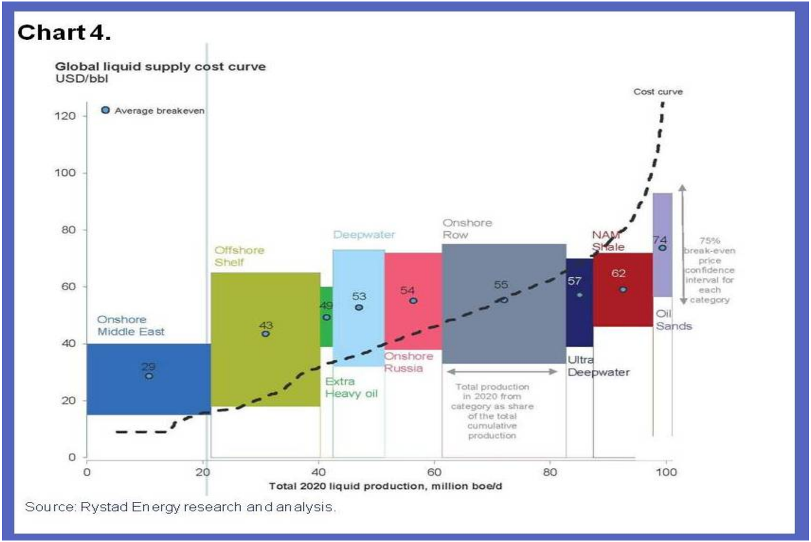


Figure: *Estimated oil extraction costs from varying sources.*

Energy Production

- Key features: supply competition between heterogeneous producers; investment in exploration and research in new tech.
- Long-running concerns about
  - dwindling fossil fuel reserves (‘peak oil’); Hotelling (1931).
  - climate change, fueling transition to sustainable energy sources.
- Oligopoly models start from a competitive view of an idealized global energy market, in which *game theory* describes the outcome of competition.
- Game is in a Cournot framework: players choose quantities of production and prices are determined by total supply.
- Reasonable for energy production: major players determine their output relative to their production costs.

Game Changers

- Start with static, or one-period games to illustrate blockading.
- The nature of the complexities calls for a dynamic model in which there are
  - dwindling reserves of oil or coal, ramping up their scarcity value;
  - discoveries of new oil reserves (over 30 major finds in 2009);
  - technological innovation such as *fracking*;
  - government subsidies for renewables such as solar and wind;
  - varying costs of production,
- These phenomena are unpredictable and dramatic: requiring *stochastic models*, with significant 'jumps' (for instance in costs or reserves).
- Some of these issues can be analyzed using the *computational tractability* of continuum mean field games.

Some References

	# Players	Type	Demand	Randomness	Replenish
Hotelling (1931)	1	–	linear	Determ.	No
Dasgupta and Stiglitz (1981)	N	Cournot	constant	single-shock	No
Deshmukh and Pliska (1983)	1	–	regimes	Poisson	Yes
Benchekroun (2008)	N	Cournot	linear	Determ.	Yes
Benchekroun et al (2009)	N	Cournot	linear	Determ.	Yes
Harris et al (2010)	1+g	Cournot	linear	Brownian	No
Ludkovski and Sircar (2011)	1+g	Cournot	linear	Poisson	Yes
Ledvina and Sircar (2012)	1+N	Bertrand	linear	Determ.	No
Ludkovski and Yang (2014)	1+g	Cournot	linear	Poisson	Yes
Colombo and Labrecciosa (2013)	N	Cournot	linear	Determ.	Yes
Dasarathy and Sircar (2014)	1+N	Cournot	linear	Poisson	Yes
Guéant et al (2010)	∞	Cournot	CES	Determ.	No
Chan and Sircar (2014)	∞	Bertrand	linear	Brownian	No

Figure: From survey paper *Game Theoretic Models for Energy Production* with M. Ludkovski in Fields Communications Volume : Commodities, Energy and Environmental Finance (2015)

Static Cournot Continuum Mean Field Game

- Market is specified by a decreasing linear inverse demand curve: $P(Q) = 1 - \text{quantity}$.
- Continuum of oil producers labelled by "position" x and density $m(x)$.
- The producer at position x has cost of production $c(x)$ per unit.
- There is an alternative energy producer with cost c_0.
- Oil producers choose $q(x)$, alternative player chooses $\hat{q}$ to solve

$$\max_{q \geq 0} q\left(1 - q - Q - \hat{q} - c(x)\right), \qquad \max_{\hat{q} \geq 0} \hat{q}(1 - \hat{q} - Q - c_0),$$

in the sense of Nash equilibrium, where

$$Q = \langle q \rangle := \int q(x) m(x)\, dx.$$

Static CMFG Blockading

- If an interior max:

$$q^*(x) = \frac{1}{2}\left(1 - Q - \hat{q} - c(x)\right), \qquad \hat{q}^* = \frac{1}{2}(1 - Q - c_0).$$

- Integrating against m and solving for Q yields

$$Q = \frac{1}{3}\left(1 - \hat{q}^* - \langle c \rangle\right), \qquad \Rightarrow \qquad Q = \frac{1}{5}\left(1 + c_0 - 2\langle c \rangle\right).$$

  Consequently,

$$\hat{q}^* = \frac{1}{5}(2 - 3c_0 + \langle c \rangle), \qquad q^*(x) = \frac{1}{5}\left(1 - \frac{5}{2}c(x) + c_0 + \frac{1}{2}\langle c \rangle\right),$$

  so $\hat{q} \geq 0$ only if $c_0 \leq \frac{1}{3}(2 + \langle c \rangle)$.

- Else blockading– alternative producer is out and

$$Q = \frac{1}{3}\left(1 - \langle c \rangle\right), \qquad \hat{q} = 0.$$

Keeping the Alternative Source Out

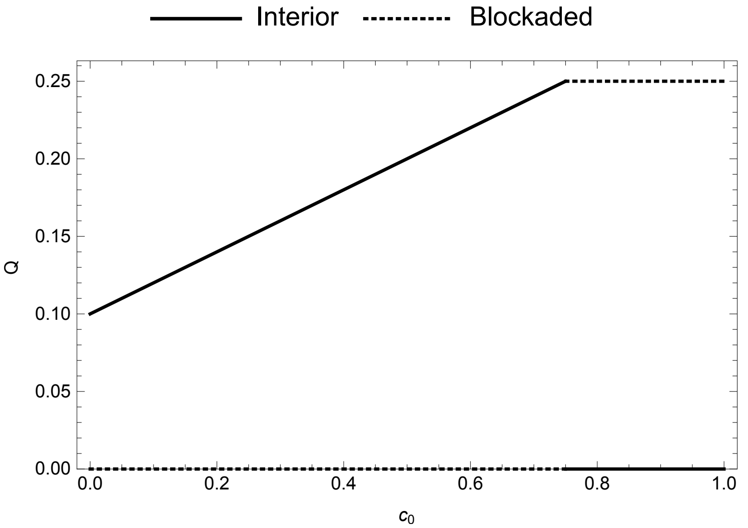


Figure: *Static CMFG: oil output Q vs. alternative cost.*

Dynamic Competition with an Expensive Energy Source

- Focus here on the exhaustible and cheap (zero cost) ‘old’ oil versus a more expensive and inexhaustible source:
- Renewables (solar) or shale oil (over short time scales). “$\infty + g$” (Major-Minor player game)
- Continuum of oil producers with initial density $M(x)$ of reserves, $x > 0$, $\int M = 1$.
- Remaining reserves $x(t)$ follow $\frac{dx}{dt} = -q_t$, where $q_t = q(t, x(t))$ is rate of production at time t, and $x(t)$ is absorbed at zero.
- $m(t, x)$ is the density of oil reserves at time t, and mean oil production rate is

$$Q(t) = \int_{\mathbb{R}_+} q(t,x)m(t,x)\,dx.$$

- The alternative player produces from a source which is expensive but abundant: marginal cost is $c > 0$; rate of production is $\hat{q}(t)$.

Value Functions

- The price received by the exhaustible producer producing $q(t,x)$

$$p(t,x) = 1 - q(t,x) - \hat{q}(t) - Q(t),$$

while for the renewable producer the price is

$$\hat{p}(t) = 1 - \hat{q}(t) - Q(t).$$

- Oil producer starting at $x(t) = x$, producing at zero cost:

$$v(t,x) = \sup_{q \geq 0} \int_t^\infty e^{-r(s-t)} q(s, x(s)) p(s, x(s))_{\{x(s)>0\}} \, ds.$$

- Alternative inexhaustible energy producers:

$$\begin{aligned} g(t) &= \sup_{\hat{q} \geq 0} \int_t^\infty e^{-r(s-t)} \hat{q}(s)(\hat{p}(s) - c)_{\{\eta(s)>0\}} \, ds \\ &\quad + \int_t^\infty e^{-r(s-t)} \frac{1}{4}(1-c)^2_{\{\eta(s)=0\}} \, ds, \end{aligned}$$

and $\eta(t) = \int_{\mathbb{R}_+} m(t,x) \, dx$, fraction of oil producers with reserves left.

Dynamic programming HJB equations

- For v and g:

$$\partial_t v + \sup_{q \geq 0} \left[q \left(1 - q - Q(t) - \hat{q}(t) - \partial_x v \right) \right] = rv,$$
$$g'(t) + \sup_{\hat{q} \geq 0} \left[\hat{q} \left(1 - \hat{q} - Q(t) - c \right) \right] = rg.$$

- The density $m(t,x)$ of reserves $x(t)$ follows the forward Kolmogorov (transport) equation

$$\partial_t m - \partial_x \left(q^* m \right) = 0,$$

with $m(0,x) = M(x)$. The mean production by exhaustible producer is given by

$$Q(t) = \int_{\mathbb{R}_+} q^*(t,x) m(t,x) \, dx.$$

- Solved up till endogenous time T when $\eta(T) = 0$, all oil exhausted, and $v(T,x) = 0$. Exhaustibility : $v(t,0) = 0$.

Full Equations

- If the renewable producer is not blockaded,

$$q^*(t,x) = \frac{1}{4}\left(1 - Q(t) + c - 2\partial_x v\right),$$

and $\hat{q}^*(t) = \frac{1}{2}\left(1 - Q(t) - c\right)$. The HJB equations become

$$\partial_t v + \frac{1}{16}\left(1 - Q(t) + c - 2\partial_x v\right)^2 = rv,$$

$$g'(t) + \frac{1}{4}\left(1 - Q(t) - c\right)^2 = rg.$$

- If the renewable producer is blockaded, we have $\hat{q}^* = 0$ and

$$q^*(t,x) = \frac{1}{2}\left(1 - Q(t) - \partial_x v\right).$$

In this case the HJB equations become

$$\partial_t v + \frac{1}{4}\left(1 - Q(t) - \partial_x v\right)^2 = rv, \qquad g'(t) = rg.$$

- With a finite number of exhaustible players, even in the two-player case, these equations are hard to handle numerically.

Numerical Solution

- Start with an initial guess Q^0 for the mean production. Then for $n = 1, 2, \ldots$:
- Step 1. Given Q^{n-1} solve the HJB equations numerically:
  (a) The optimal strategy of the renewable producer is

$$\hat{q}^n(t) = \frac{1}{2}\left(1 - Q^{n-1}(t) - c\right)^+ .$$

  (b) The exhaustible producer solves the optimal control problem

$$\partial_t v^n + \frac{1}{4}\left(1 - Q^{n-1}(t) - \partial_x v^n\right)_{bl}^2 + \frac{1}{16}\left(1 - Q^{n-1}(t) + c - 2\partial_x v^n\right)^{2\,c}_{bl} = r v^n.$$

  The feedback production strategy of the exhaustible producer is

$$q^n(t,x) = \frac{1}{2}\left(1 - Q^{n-1}(t) - \partial_x v^n\right)_{bl} + \frac{1}{4}\left(1 - Q^{n-1}(t) + c - 2\partial_x v^n\right)^c_{bl}.$$

Numerical Solution *(ctd.)*

- Step 2. Given q^n, solve the forward Kolmogorov equation

$$\partial_t m^n - \partial_x [m^n q^n] = 0.$$

This gives the aggregate production Q^n for the next iteration

$$Q^n(t) = \int_{\mathbb{R}+} q^n(t,x) m^n(t,x)\, dx.$$

- Better to consider the tail distribution function

$$\eta(t,x) = \int_x^\infty m(t,y)\, dy.$$

which solves

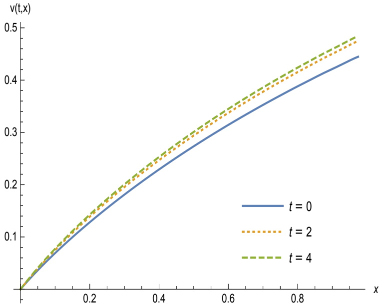
$$\partial_t \eta(t,x) - q(t,x)\partial_x \eta(t,x) = 0,$$

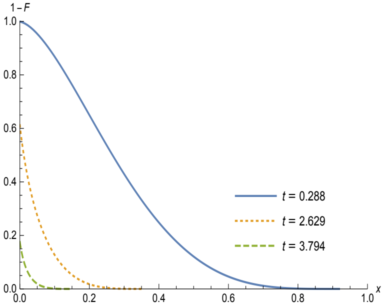
with initial condition $\eta(0,x) = \int_x^\infty M(y)\, dy$ (can handle point masses).

- Initialization of iterative algorithm: use the monopoly problem where the value function can be computed explicitly.

Numerical Results

- Base case parameters: $r = 0.2, M \sim \text{Beta}(2,4)$ and $c = 0.9$. Iterative algorithm converges rapidly, typically within 10 iterations.





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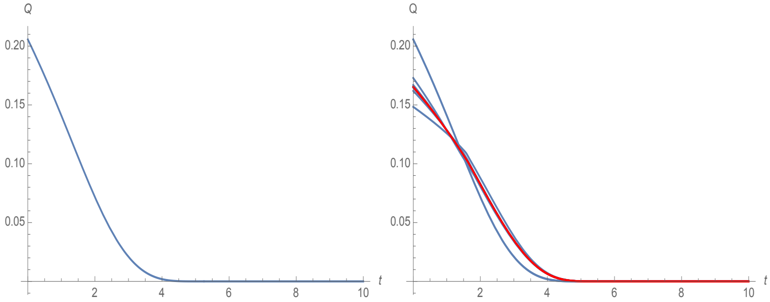

Notice that the exhaustible producers slow down production in the presence of a renewable/alternative competitor.

Blockading of renewable/alternative producer

When c is high and oil is plentiful, energy price too low for alternative producer to enter the market. He is blockaded. Here $c = 0.9$.

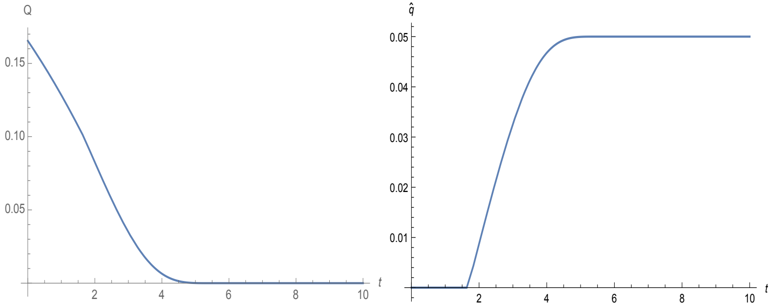

Figure: Production rates for the exhaustible (left) and renewable (right) producers. Notice the renewable producer is blockaded until about $t = 1.5$.

Strategic blockading against entry of alternative resources

When c is high enough, the oil producers may strategically increase their aggregate production in the short run to keep the alternative energy out of the market.

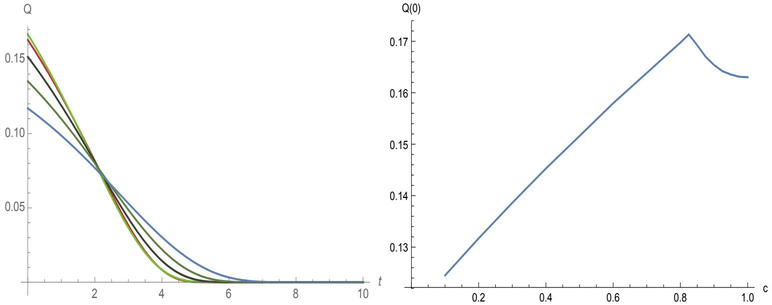

Figure: Left panel: the aggregate production rate Q for 5 different values of production costs $c = 0, 0.2, 0.4, 0.6, 0.8$. Right panel: initial production rate of the exhaustible producers. Notice the strategic blockading of entry for large c.

Existence & Uniqueness Theory

- General analysis of mean field games (e.g. Lasry-Lions '07, Huang-Malhame-Caines '06, Bensoussan-Frehse-Yam '13, Cardaliaguet *et al.* '12-'15) deals with systems of the form

$$v_t + \frac{1}{2}\sigma^2 v_{xx} - rv + H(t, x, v_x) = V[m],$$
$$m_t - \frac{1}{2}\sigma^2 m_{xx} - (G(t, x, v_x)m)_x = 0,$$

  where $V[m]$ is a monotone operator.
- In the case of oligopoly models, the coupling happens nonlocally in the Fokker-Planck equation:

$$v_t + \frac{1}{2}\sigma^2 v_{xx} - rv + H(t, v_x, [mv_x]) = 0,$$
$$m_t - \frac{1}{2}\sigma^2 m_{xx} - (G(t, v_x, [mv_x])m)_x = 0.$$

Existence & Uniqueness Theory

- In those models, interaction is through the mean of the state $\int xm$ (controlled McKean-Vlasov systems).
- For us, it is through the mean of the controls $\int qm$.
- The boundary condition at $x = 0$ (exhaustibility) is not addressed in the bulk of this theory, and the tractable examples on the full space are linear-quadratic MFGs.
- However, recently, Graber & Bensoussan '15 prove an existence and uniqueness theorem for a classical solution of this oligopoly system on a finite domain $[0, T] \times [0, L]$, basically a "hard analysis" for this specific problem.

Exploration and Random Discoveries

- So far: exhaustibility or scarcity leads to price increases/shocks – INTERMEDIATE.
- However, proven reserves of crude oil rose 13% to 25.2 billion barrels in 2010.
- Multiple discoveries resulted in reasonably stable oil prices in the ’80s.
- We analyze effect of exploration and random discoveries in a dynamic Cournot (continuum) game – SHORT TERM.

Resource Discovery

- The remaining reserves X of the oil producers follow

$$dX_t = -q_t \, \mathbb{1}_{\{X_t>0\}} \, dt + \delta \, dN_t,$$

  where (N_t) is a controlled counting process with intensity λa_t, penalized by convex cost function $\mathcal{C}(a_t)$.
- Each discovery leads to an increase in reserves by $\delta > 0$.
- Here we consider only oil producers with zero extraction costs (no alternative producers):

$$v(t,x) = \sup_{q,a} \mathbb{E} \left\{ \int_t^\infty e^{-r(s-t)} \left\{ q_s p_{s\{X_s>0\}} - \mathcal{C}(a_s) \right\} ds \,\middle|\, X_t = x \right\}.$$

- Take power costs

$$\mathcal{C}(a) = \frac{1}{\beta} a^\beta + \kappa a, \qquad \beta > 1, \kappa \geq 0.$$

  This guarantees a finite *saturation point* $x_{sat} < \infty$ such that $a^*(x) = 0$ for $x > x_{sat}$, and (X_t) does become arbitrarily large infinitely often.
- Numerical stationary solution to the mean field game:

Stationary Solution

Look for stationary solution to the mean field game equation system:

$$
\begin{aligned}
rv(x) &= \sup_{q\geq 0}\left\{q\left(1-q-\epsilon Q-v'(x)\right)\right\}+\sup_{a\geq 0}\left\{a\lambda\Delta v-\mathcal{C}(a)\right\},\\
0 &= -\frac{d}{dx}\left(q^*(x)m(x)\right)-\lambda\left\{a^*(x-\delta)m(x-\delta)-a^*(x)m(x)\right\},\\
a^*(x) &= \left(\mathcal{C}'\right)^{-1}\left(\lambda\Delta v(x)\right),\quad q^*(x)=\frac{1}{2}\left(1-\epsilon Q-v'(x)\right),\\
Q &= \int_{\mathbb{R}_+} q^*(x)m(x)\,dx.
\end{aligned}
\tag{1}
$$

with ‘revival’ boundary condition

$$
v(0)=\sup_{a\geq 0}\mathbb{E}\left[e^{-r\tau}v(\delta)-\int_0^\tau e^{-rt}\mathcal{C}(a)\,dt\right]=\sup_{a\geq 0}\frac{a\lambda v(\delta)-\mathcal{C}(a)}{\lambda a+r}.
$$

Numerical Stationary Solution

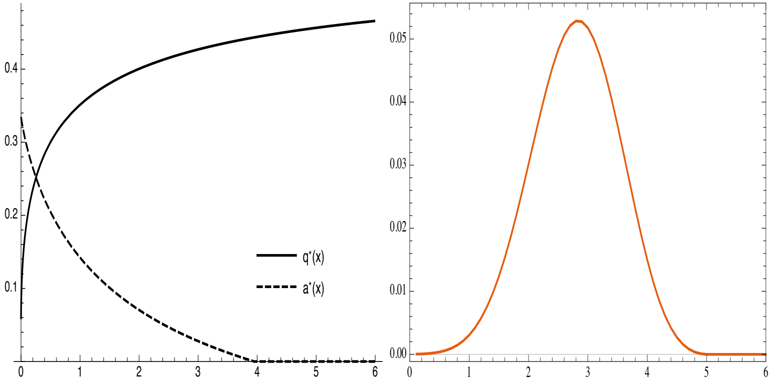

Figure: The parameters are $\delta = 1, \lambda = 1, r = 0.1, \mathcal{C}(a) = 0.1a + a^2/2$ and $\epsilon = 0.25$.

Sample Game Path

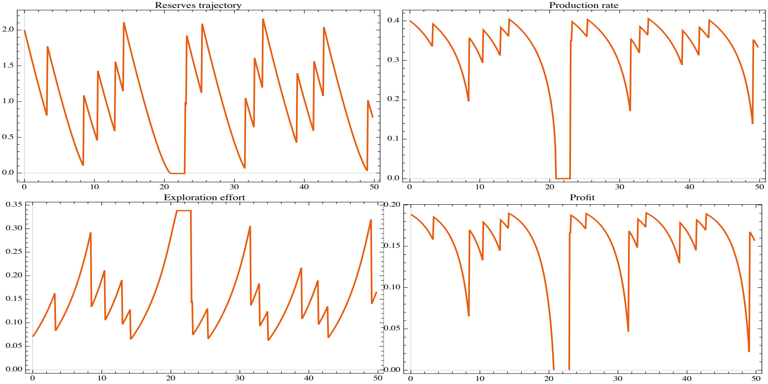

Figure: Trajectory of the game solution over time.

Ongoing Directions

- Incorporating cost curves: continuum of exhaustible producers with costs $c(x)$ *vs.* continuum ($z \in [0, 1]$) of renewables with costs $s(z)$.
- Most of the Cournot energy analyses have a fixed pricing (inverse demand) curve $P(Q)$, typically $P(Q) = 1 - Q$.
- But recent (2015-16) failed rallies in oil price could be due to uncertainty about China's demand for oil:
  - it grew 6-fold from 2003 to 2013;
  - it accounted for 45% of total growth in oil demand in that time.
- In 2015: China GDP growth 7.3%, slowest since 1990.
- Stochastic demand: take $P = Y_t - Q$, where Y is China, India , Iran, ... MFG in random environment.
- Electricity markets, bid-stack, producers bid supply curves $p_i(q_i)$. Supply function equilibrium problems.