Control of Aggregate Electric Water Heating Loads via Mean Field Games Based Methods

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Advances in Modelling and Control for Power Systems of the Future
Palaiseau, Sep 2018

Practical context: The changing role of load management

- Initial idea: Using deferrable grid electric loads, mainly thermal loads (electric space heating, air conditioners, refrigerators) to shave electric demand peaks and fill valleys
  - deferred generation and transmission system expansion, efficiency of electricity production
- Currently: Strong push for renewable sources
  - Intermittency
  - A control architecture to convert dispersed grid thermal loads into an effective energy storage potential
- Challenges: Literally millions of devices to (in theory) observe and actuate
  - staggering communication load, staggering computations, challenge to keep customers happy

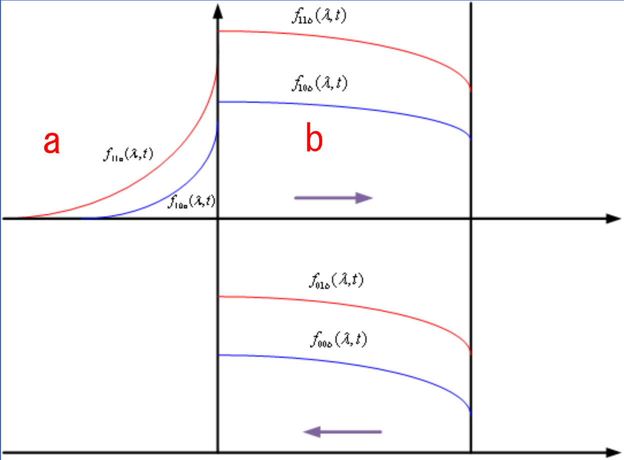
Past Approach: Homogeneous deterministic direct load control

- Idea: Send the same interruption/reconnection signal to very large numbers of homogeneous electric devices attached to thermostatically controlled loads.
- Analytical tools (statistical mechanics): Microscopic level stochastic models under uniform control + Law of large numbers
  - Ensemble statistics (PDE description)
- An example (Laurent - Malhamé 1994): Elemental hybrid state (continuous temperature, discrete thermostat) stochastic model of water heaters with Kolmogorov equations based wave model at the aggregate level.
- Main problem: Uniform controls applied to devices in potentially very different states.
  - Too careful $\rightarrow$ too conservative.
  - Too daring $\rightarrow$ a fraction of customers unhappy.

Improvements since

- Higher order PDE model extensions: (Zhao, Zhang 2017)
- PDE based randomized controls locally implemented (Totu, Winieski 2017)
- PDE based identification approaches (Moura, Bendsten, Ruiz 2014)
- Aggregation and state estimation based control (Mathieu, Koch, Callaway 2013)
- Mean field related ideas (Meyn, Barooah, Bušić, Ehren 2013)
- Mean field game based methods (Kizilkale, Malhamé 2013, Ma, Callaway and Hiskens 2013)

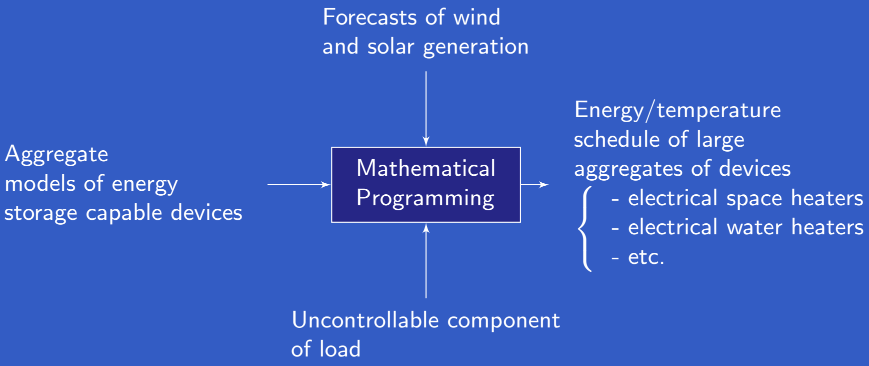
Aggregate PDE model of electric water heaters (1994)



Implementation principles

1. Decentralization: Each controller has to be situated locally because comfort and safety constraints can be locally secured.
2. Parsimony of communications: Communications should be kept at minimum both with the central authority and among users.
3. Minimal intrusiveness of controls: Deviations from uncontrolled behavior should be minimized.

Envisioned Overall Architecture: The case of a single central authority



MFG = (Statistical Mechanics + Optimal Control Theory)

Linear Quadratic Mean Field Rendez-vous Problem

$$J_i(u^i, u^{-i}) = \mathbb{E}\int_0^\infty e^{-\delta t}\left\{\left[x_t^i - \gamma\left(\bar{x}^N + \eta\right)\right]^2 q + \left(u_t^i\right)^2 r\right\} dt, \quad 1 \le i \le N$$

MFG's: A Fast Growing Literature

Controls and PDEs:

Huang M. Y., Malhame R. P. and Caines P. E. (2006), “Large population stochastic dynamic games: closed-loop McKean-Vlasov systems and the Nash certainty equivalence principle”. Communications in Information and Systems. Vol. 6, No. 3, pp. 221-252.

Lasry J. M. and Lions, P. L. (2006), “Jeux à champ moyen. I-Le cas stationnaire”. Comptes Rendus Mathematique. Vol. 343, No. 9, pp. 619-625.

Lasry J. M. and Lions, P. L. (2006), “Jeux à champ moyen. II-Horizon fini et contrôle optimal”. Comptes Rendus Mathematique. Vol. 343, No. 10, pp. 679-684.

Huang M. Y., Caines P. E. and Malhamé R. P. (2007), “Large population cost-coupled LQG problems with non-uniform agents: individual-mass behaviour and decentralized ϵ-Nash equilibria”. IEEE Tans. on Automatic Control, Vol. 52, No. 9, pp. 1560-1571.

Books:

Bensoussan A., Frehse J., Yam P. (2013), "Mean Field Games and Mean Field Type Control Theory" Carmona R.,

Delarue F. (2018), "Probabilistic Theory of Mean Field Games with Applications I and II"

Mean Field Games: The Reasons?

Two fundamental reasons:

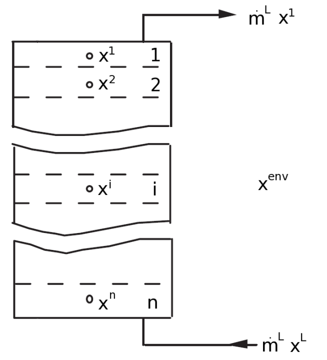
- Games are a natural device for enforcing decentralization.
- The large numbers involved induce decoupling effects which allow the law of large numbers to kick in.

Practical benefits:

- The resulting control laws can be computed in an open loop manner by individual devices thus significantly reducing communication requirements.
- Control implementation is local unlike direct control, thus permitting local enforcement of comfort and safety constraints.

Non-cooperative Collective Target Tracking Mean Field Control for Water Heaters

Water Heater Stratification Model



x_l	temperature of the lth segment
u_l	control action at the lth segment
$\dot{m}_L$	fluid mass flow rate to the load
$\dot{Q}_l$	rate of energy input by the heating element to
x^{env}	temperature of the environment
x^L	temperature of the inlet fluid
M_l	mass of the fluid in the lth segment
A_l	lateral surface area of the lth segment
C^{pf}	specific heat of the fluid
U	loss coefficient between the tank and its enviror

$$M_l C^{pf} \frac{dx_{l,t}}{dt} = UA_l(x^{env} - x_{l,t}) + \dot{m}_t^L C^{pf}(x_{(l+1),t} - x_{l,t}) + \dot{Q}_l u_{l,t},$$
$$t \geq 0, \quad l \neq n$$

$$M_l C^{pf} \frac{dx_{l,t}}{dt} = UA_l(x^{env} - x_{l,t}) + \dot{m}_t^L C^{pf}(x_t^L - x_{l,t}) + \dot{Q}_l u_{l,t},$$
$$t \geq 0, \quad l = n$$

S. Klein, "A design procedure for solar heating systems," Ph.D. dissertation, Department of Chemical Engineering, University of Wisconsin-Madison, 1976.

Elemental Agent Dynamics

- $\dot{m}^L$: modeled as a (stochastic) jump process.
- Physical model:

$$M_l C^{pf} \frac{dx_{l,t}}{dt} = UA_l(x^{env} - x_{l,t}) + \dot{m}_t^L C^{pf}(x_{(l+1),t} - x_{l,t}) + \dot{Q}_l u_{l,t},$$
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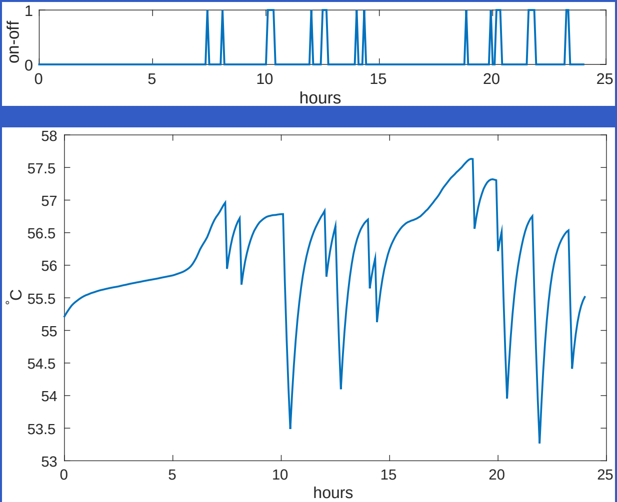
$$M_l C^{pf} \frac{dx_{l,t}}{dt} = UA_l(x^{env} - x_{l,t}) + \dot{m}_t^L C^{pf}(x_t^L - x_{l,t}) + \dot{Q}_l u_{l,t},$$
$$t \geq 0, \quad l = n$$

- We write it as:

$$\frac{dx_t}{dt} = A^{\theta_t} x_t + B u_t + c^{\theta_t}, \quad t \geq 0.$$

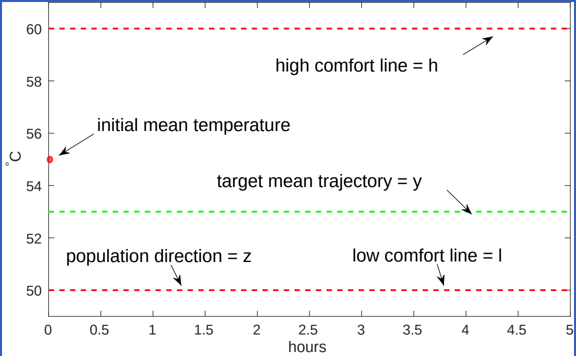
- θ_t, $t \geq 0$, is a continuous time Markov chain taking values in $\Theta = \{1, 2, ..., p\}$ with infinitesimal generator matrix Λ. Each discrete value is associated with a type of event (showers, dishwashers, etc ...)

Sample Trajectory



A Single Agent's Temperature Trajectory Applying Markovian Jump LQ Tracking

Constant Level Tracking Problem Setup



Redefined Dynamics

Dynamics for a population of N water heaters:

$$\frac{dx_t^i}{dt} = A'^{\theta_t^i} x_t^i + B u_t^i + c'^{\theta_t^i}, \quad t \geq 0, \quad 1 \leq i \leq N$$

The control input is redefined so that no control effort is required on average to remain at initial temperature.

$$\frac{dx_t^i}{dt} = A^{\theta_t^i} x_t^i + B(u_t^i + u_t^{i,free}) + c^{\theta_t^i}, \quad t \geq 0, \quad 1 \leq i \leq N,$$

where

$$u_m^{i,free} = \sum_{l=1}^{n} UA_l(x_{l,0}^i - x^{env}) + \mathbb{E} \sum_{j=1}^{p} \zeta^j(t) \dot{m}_t^L(j) C^{pf}(x_{1,t} - x_t^L)$$

$\zeta(t) = [\zeta^1(t), ..., \zeta^p(t)]$ is the probability distribution of the Markov chain

Integral Control Based Cost Function

Cost functions:

$$J_i^N(u^i, u^{-i}) = \mathbb{E} \int_0^T \left[(Hx_t^i - z)^2 q_t^y + (Hx_t^i - Hx_0^i)^2 q^{x_0} + \|u_t^i\|_R^2 \right] dt$$
$$+ (Hx_T^i - z)^2 q_T^y + (Hx_T^i - Hx_0^i)^2 q^{x_0}$$

x^i	temperature
z	lower comfort bound
u^i	control
H	$[1/n, ..., 1/n]$

Integral controller embedded in mean-target deviation coefficient: q_t^y, $t \in [0, T]$, calculated as the following integrated error signal:

$$q_t^y = \left| \lambda \int_0^t (H\bar{x}_\tau^N - y) d\tau \right|$$

$\bar{x}^N$	mean temperature of the population
y	mean target

Synthesis of the Mean Field Control Law: Step 1

For a given $\bar{x}_t$ and thus q_t^y, $t \in [0, T]$, compute optimal agent response: [W. M. Wonham, 1971]

- each agent $\mathcal{A}_i, 1 \leq i \leq N$, obtains the positive solution to the coupled set of Riccati equations

$$-\frac{d\Pi_t^j}{dt} = \Pi_t^j A^j + {A^j}^\top \Pi_t^j$$
$$- \Pi_t^j B R^{-1} B^\top \Pi_t^j + \sum_{k=1}^{p} \lambda_{jk}(t) \Pi_t^k + (q_t^y + q^{x_0}) H^\top H,$$

where

$$\Pi_T^j = (q_T^y + q^{x_0}) H^\top H, \quad 1 \leq j \leq p$$

Synthesis of the Mean Field Control Law: Step 1

- for a given target signal z, the individual ith agent offset function is generated by the coupled differential equations

$$-\frac{ds_{i,t}^j}{dt} = (A^j - BR^{-1}B^\top \Pi_t^j)^\top s_{i,t}^j - q_t^y H^\top z - q^{x_0} H^\top x_0^i + \Pi_t^j c_i^j + \sum_{k=1}^{p} \lambda_{jk}(t) s_{i,t}^k,$$

where

$$s_{i,T}^j = -[q_T^y H^\top z + q^{x_0} H^\top x_0^i], \quad 1 \leq j \leq p$$

- the optimal tracking control law is given by

$$u_{i,t}^\circ = -\sum_{j=1}^{p} I_{[\theta_{i,t}=j]} R^{-1} B^\top (\Pi_t^j x_{i,t} + s_{i,t}^j), \quad t \geq 0.$$

Fixed Point Equation System: Step 2

Under best response to posited $\bar{x}_t$, agents mean must replicate $\bar{x}_t$.

$$q_t^y = \left|\lambda \int_0^t (H\bar{x}_\tau - y)d\tau\right|,$$

$$-\frac{d\Pi_t^j}{dt} = \Pi_t^j A^j + A^{j\top}\Pi_t^j - \Pi_t^j B R^{-1} B^\top \Pi_t^j$$
$$+ \sum_{k=1}^p \lambda_{jk}\Pi_t^k + (q_t^y + q^{x_0})H^\top H, \quad \Pi_T^j = (q_T^y + q^{x_0})H^\top H, \quad 1 \le j \le p,$$

$$-\frac{ds_t^j}{dt} = (A^j - BR^{-1}B^\top \Pi_t^j)^\top s_t^j - q_t^y H^\top z - q^{x_0} H^\top \bar{x}_0$$
$$+ \Pi_t^j c^j + \sum_{k=1}^p \lambda_{jk} s_t^k, \quad s_T^j = -[q_T^y H^\top z + q^{x_0} H^\top \bar{x}_0], \quad 1 \le j \le p,$$

$$\frac{d\bar{x}_t^j}{dt} = (A^j - BR^{-1}B^\top \Pi_t^j)\bar{x}_t^j + \sum_{k=1}^p \lambda_{kj}\bar{x}_t^k + \zeta_t^j c^j - \zeta_t^j BR^{-1}B^\top s_t^j, \quad 1 \le j \le p$$

$$\bar{x}_t = \sum_{j=1}^p \bar{x}_t^j,$$

$$\frac{d\zeta_t}{dt} = \zeta_t \Lambda^\top.$$

One recalls

- $\bar{x}_t^j = E(\bar{x}_t I_{[\theta_t = j]})$
- $\Lambda = \{\lambda_{i'j'}, i', j' = 1, ..., p\}$ is the infin. gen. of the MC
- $\zeta_t = [\zeta_t^1, \ldots, \zeta_t^p]$ is the prob. dist. of the MC

Fixed Point Analysis

Define the set $\mathcal{G}$: all func. s.t. $f \in \mathbf{C}_b[0,T], f(0) = \bar{x}_0$ and $z \leq f(t) \leq \bar{x}_0, t \in [0,T]$.

Lemma: Under $\|\cdot\|_\infty$, $\mathcal{G}$ is closed in $\mathbf{C}_b[0,T]$; therefore it is a complete metric space.

Theorem: The assumption below guarantees the existence of a unique fixed point for the map $\mathcal{M} : \mathcal{G} \to \mathcal{G}$, due to the Banach fixed point theorem.

Contraction Assumption

$$\frac{2b_1}{a_1 \min_j \sqrt{\inf_{(0\leq t\leq T, \bar{x}\in\mathcal{G})}[\lambda_{min}(\Pi_t^j)]}} < 1$$

where

$$a_1 = \min_j \frac{\inf_{(0\leq t\leq T,\bar{x}\in\mathcal{G})}\left[\lambda_{min}(Q_t^j + \Pi_t^j BR^{-1}B^\top \Pi_t^j)\right]}{\sup_{(0\leq t\leq T,\bar{x}\in\mathcal{G})}\left[\lambda_{\sup}(\Pi_t^j)\right]},$$

$$b_1 = \max_j \sqrt{\sup_{(0\leq t\leq T,\bar{x}\in\mathcal{G})}\left[\lambda_{max}(\Pi_t^j)\right]}\|BR^{-1}B^\top\|(\gamma_1+\gamma_2+\gamma_3+\gamma_4),$$

$$\gamma_1 = (\max_j \kappa_3^j \lambda)\left[(\sqrt{p/n}q^{x_0}\bar{x}_0) + (\sqrt{p/n}\kappa_1 z)\right],$$

$$\gamma_2 = \lambda\sqrt{p/n}z \sup_{\bar{x}\in\mathcal{G}}\left\|\Psi^{\bar{x}}(t,T)\right\| T,$$

$$\gamma_3 = (\max_j \kappa_2^j \lambda)(\sqrt{p/n}q^{x_0}\bar{x}_0)(\sqrt{p/n}\kappa_1 z)(c_j \sup_{(0\leq t\leq T,\bar{x}\in\mathcal{G})}[\lambda_{max}(\Pi_t)]),$$

$$\gamma_4 = (\sqrt{p/n}z + \sqrt{n}\max_j\|c^j\|\kappa_2^j)\lambda \int_T^t \sup_{\bar{x}\in\mathcal{G}}\left\|\Psi^{\bar{x}}(t,\tau)\right\|\tau d\tau,$$

where $\dfrac{d\Psi^{\bar{x}}(t,\tau)}{dt} = -[G^\top + \Lambda\otimes I]\Psi^{\bar{x}}(t,\tau),$

$G = diag(G_1,\ldots,G_p)$, and $G_j = A_j - BR^{-1}B^\top\Pi^j$.

ϵ-Nash Theorem

Collective Target Tracking MJ MF Stochastic Control Theorem

Under technical conditions the Collective Target Tracking MJ MF Equations have a unique solution which induces a set of controls $\mathcal{U}_{col}^N \triangleq \{(u^i)^0; 1 \leq i \leq N\}$, $1 \leq N < \infty$, with

$$u_t^\circ = -\sum_{j=1}^{p} I_{[\theta_t = j]} R^{-1} B^\top (\Pi_t^j x_t + s_t^j), \quad t \geq 0,$$

such that

1. all agent system trajectories x^i, $1 \leq i \leq N$, are second order stable;
2. $\{\mathcal{U}_{col}^N; 1 \leq N < \infty\}$ yields an ϵ-Nash equilibrium for all $\epsilon > 0$.

Control Architecture

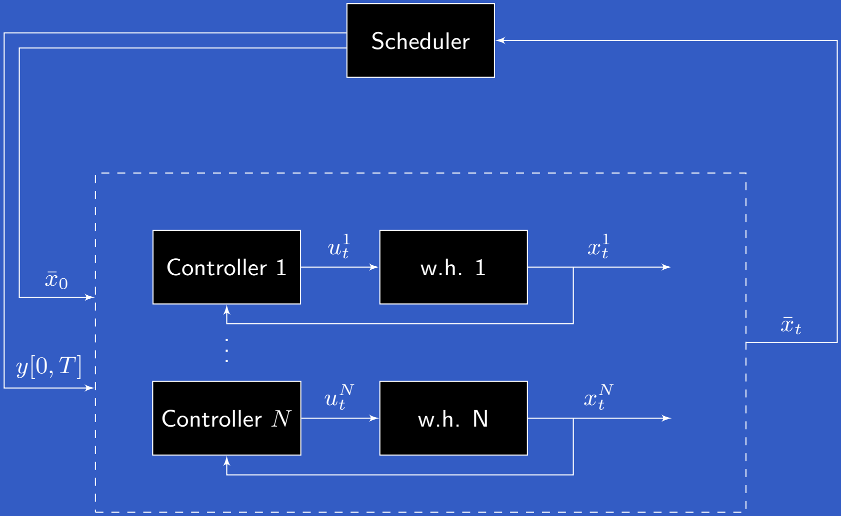


Figure: Control Architecture

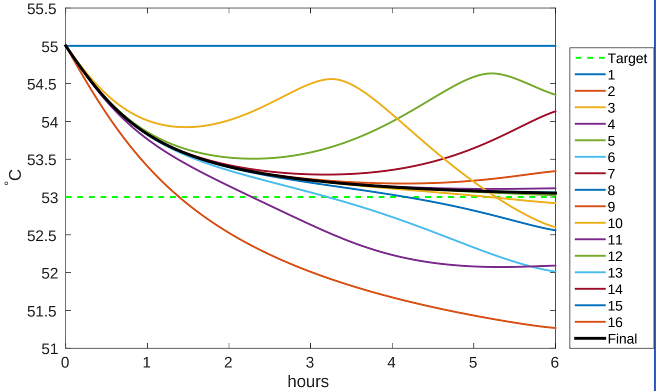
Simulations

- 200 water heaters (60 gallons): 2 stratification layers
- Two elements with total maximum elemental power of 4.5kW
- initial mean: 55°C
- 2 experiments:
    - increase 2 °C mean temperature,
    - decrease 2 °C mean temperature,

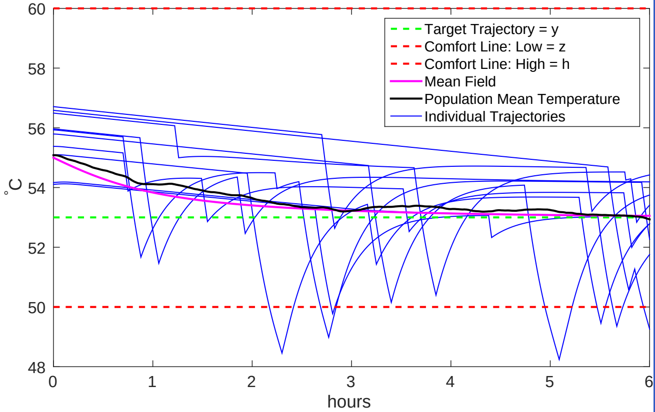
  over a 6 hours control horizon
- constant water extraction rate: 0.05 l/sec
- time invariant 2 state Markov chain:
    - arrival rate: 0.5
    - departure rate: 7
    - consequently average water consumption is 288 l/day

  The central authority provides the target, local controllers apply collective target tracking mean field

Fixed Point Iterations

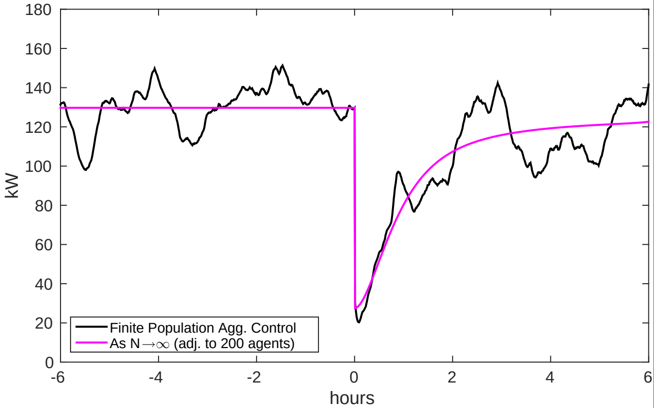


Iterations



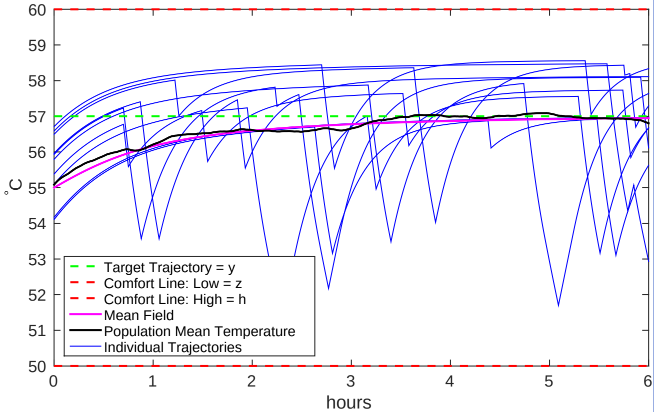
Agents Applying Collective Target Tracking Markovian Jump MF Control:
All Agents Following the Low Comfort Level Signal

Aggregate Power Relief Curve



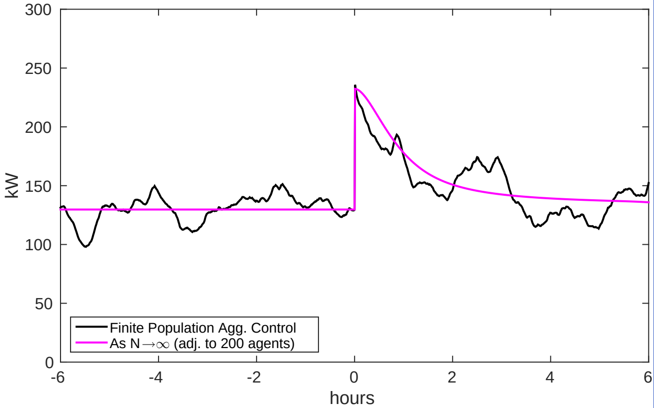
Aggregate Power Consumption

Energy Accumulation: Collective Target Tracking Markovian Jump MFC



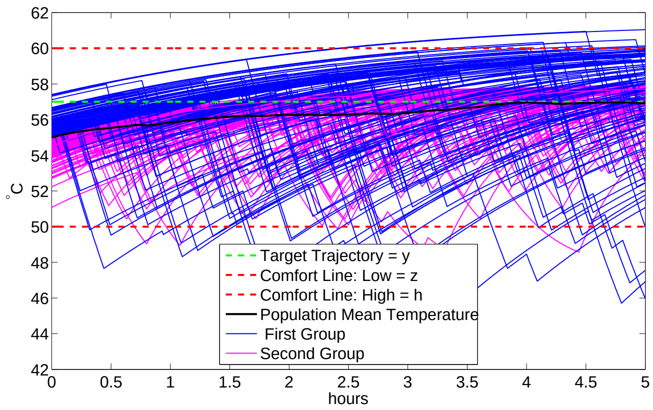
Agents Applying Collective Target Tracking Markovian Jump MF Control:
All Agents Following the High Comfort Level Signal

Aggregate Power Accumulation Curve



Aggregate Power Consumption

Energy Accumulation: Heterogeneous Populations

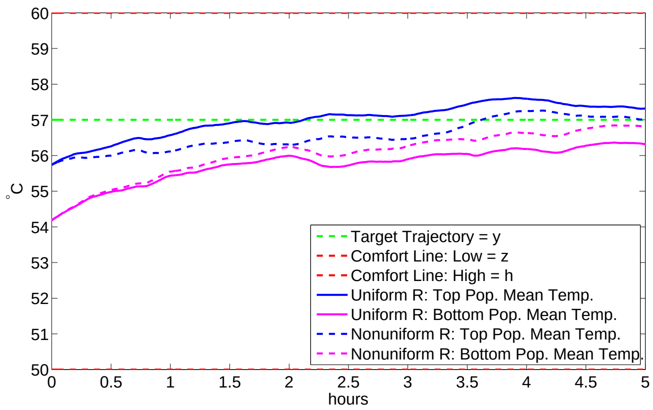


Agents Applying Collective Target Tracking Markovian Jump MF Control

First group: higher initial temperature, second group: lower initial temperature

Second group's control penalty coefficient R is lower than the first group

Energy Accumulation: Homogeneous vs Heterogeneous Populations)



First group: higher initial temperature, second group: lower initial temperature

experiment 1: same control penalty coefficient R for both groups

experiment 2: second group's control penalty coefficient R is lower than the first group

Conclusions / Future Work

- Mean field games based control is a natural approach for load management in a smart grid context.
- It exploits the predictability of large number averages to produce decentralized controls with near centralized optimality properties.
- It preserves system diversity while minimizing communications requirements.
- It is a flexible tool for shaping control effort among devices.

Weakness:

- It overly relies on a correct statistical description of the underlying driving stochastic processes as well as the random distribution of device parameters.

Future work:

- Develop online device model parameter identification and adaptation algorithms.
- Consider time varying collective target tracking problems.
- Better address the impact of local constraints on global target generation.
- Investigate cooperative mean field control solutions.