McKean FBSDE applied to the management of microgrid under uncertainty of production/consumption

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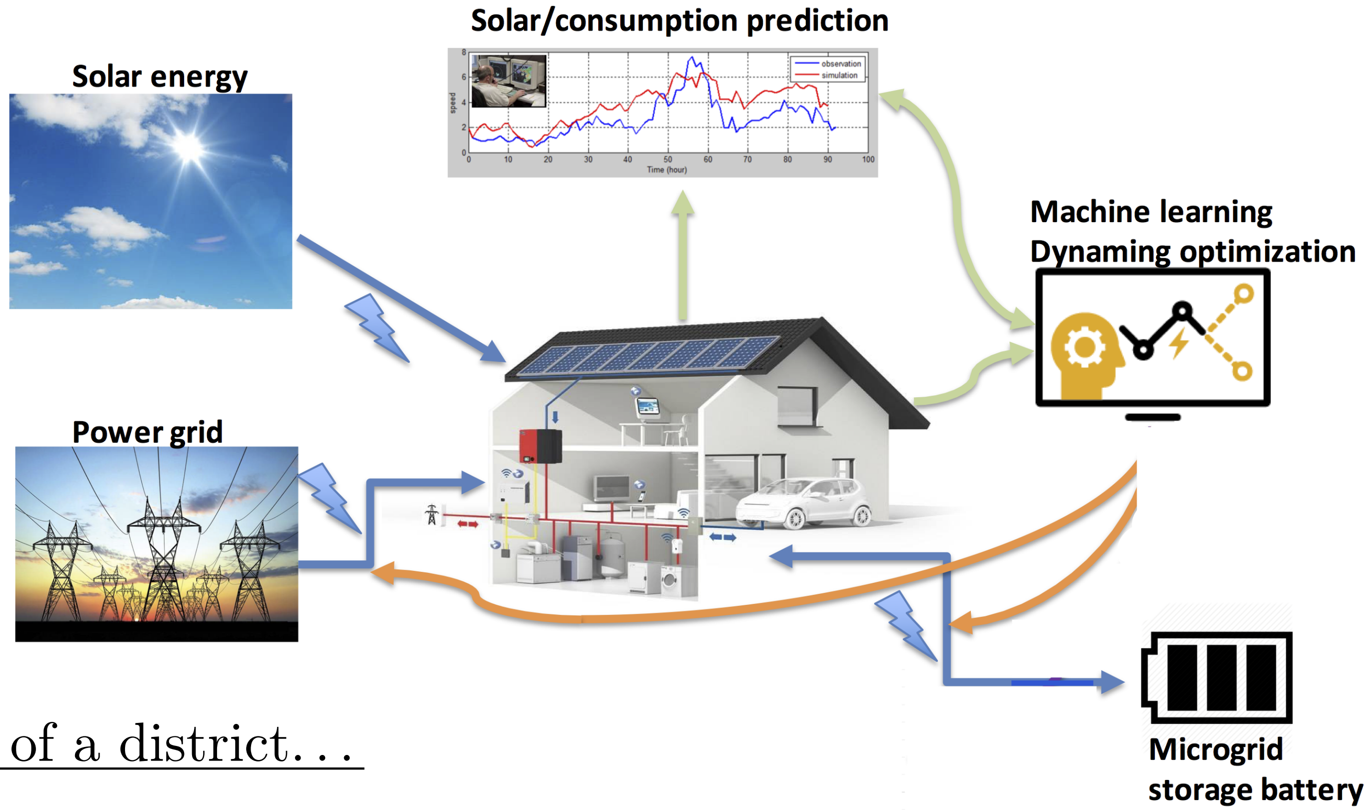
Contents

1 Modeling the micro-grid

1.1 Context

Recent deep transformations in the mechanisms of energy purchase / sale, distribution / consumption: Renewable Energies, storage, aggregation, home automation, setting up of microgrids…



At the scale of a district…

1.2 The system

- ✓ **Context:** management of smart district/building
  - equipped with solar panels (one could complete also with wind farms)
  - connected to a "public" grid providing electricity
  - equipped with a battery (or any storage capacities)
- ✓ **State variables: Grid Power, State Of Charge**, weather variables, inside temperature, building consumption
- ✓ **Uncertainty: global consumption, PV production (intermittent)**
- ✓ **Controls:** HVAC, lighting, **battery** (playing the role of a buffer)
- ✓ **Economic criterion**: **reduction of the uncertainty of the demand on the grid load, look for smoothing the demand over the day**
  - Electricity producer: better sizing of energy-production units
  - Grid manager: better management of power flow on the public grid
  - Consumer (or aggregator): contract with lower electricity price

1.3 Time scale of the optimization problem

Optimization window = 24h to 48h, to account for

✓ large variability of weather forecast (impact on PV production)

✓ variability of consumption forecast (in particular if any industrial activities in the district)

Timeline

✓ At Day D-1 before noon:

- get the weather forecast from MeteoFrance as a single point forecast
- build a probabilistic model of irradiance uncertainty (for PV production)
- compute the optimal mean consumption on the public grid for Day D [➠ McKean optimization]
- send this as a demand for electricity on the spot market

✓ At Day D:

- use the optimal strategy for battery management computed at Day D-1

✓ Time-consistency when forecasts are updated?

1.4 Ingredients for designing the micro-grid management

Goal: How to minimize the variability of the grid load?

1. **Consider an optimization criterion**

   For instance: over $T = 1$ day horizon,

$$\min_{\mathbf{control}_t} \int_0^{\mathbf{T}} \left(\kappa \, \mathbb{V}\mathrm{ar}\left[\mathtt{P}_{\mathtt{grid}}(\mathbf{t})\right] + \mu \, \mathbb{E}\left[\mathtt{P}^{\mathbf{2}}_{\mathtt{bat}}(\mathbf{t})\right] + \nu \, \mathbb{E}\left[\left(\mathtt{SOC}(\mathbf{t}) - \frac{\mathbf{1}}{\mathbf{2}}\right)^{\mathbf{2}}\right]\right) \mathrm{d}\mathbf{t}$$

$$+ \tilde{\nu} \, \mathbb{E}\left[\left(\mathtt{SOC}(\mathbf{T}) - \frac{\mathbf{1}}{\mathbf{2}}\right)^{\mathbf{2}}\right]$$

✓ Compromise between

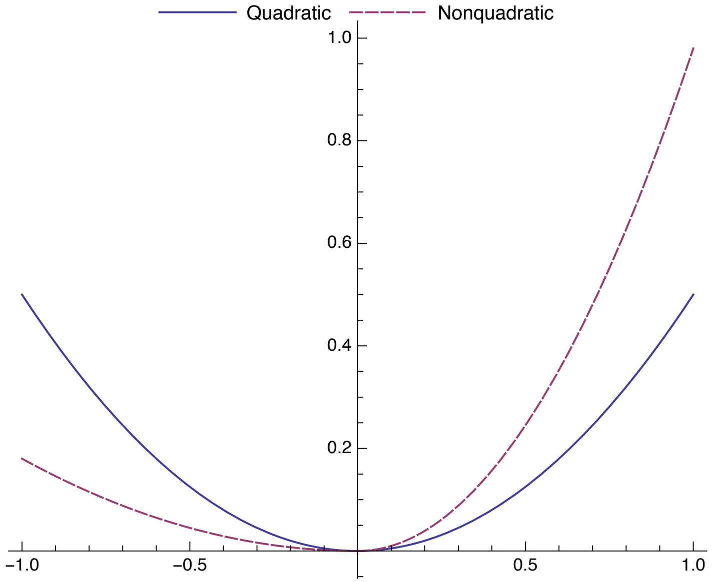
- variability of $\mathtt{P}_{\mathtt{grid}}$ averaged over the day
- large charge/discharge of the battery (aging effect)
- maintening the battery at the medium level of charge

✓ Installation cost treated separately

✓ Looks like a Linear-Quadratic problem

Variant with penalizing differently excess/deficit of demand

✓ Convex loss function ℓ:



✓ $$\min_{control_t} \int_0^T \Big(\kappa \inf_{\mathbf{m}} \mathbb{E}\left[\ell(\mathtt{P}_{\mathtt{grid}}(\mathbf{t}) - \mathbf{m})\right] + \mu\, \mathbb{E}\left[\mathtt{P}^2_{\mathtt{bat}}(t)\right] + \nu\, \mathbb{E}\left[(\mathtt{SOC}(t) - \frac{1}{2})^2\right]\Big)\mathrm{d}t + \tilde{\nu}\, \mathbb{E}\left[(\mathtt{SOC}(T) - \frac{1}{2})^2\right]$$

✓ If $\ell(x) = x^2$, $m^\star = \mathbb{E}\left[\mathtt{P}_{\mathtt{grid}}(t)\right]$ and $\inf_m \mathbb{E}\left[\ell(\mathtt{P}_{\mathtt{grid}}(t) - m)\right] = \mathbb{V}\mathrm{ar}\left[\mathtt{P}_{\mathtt{grid}}(t)\right]$.

✓ Regarding Spot market: $m^\star =$ optimal mean consumption for Day D

✓ In the following, we replace the inf by $\mathbb{E}\left[\ell(\mathtt{P}_{\mathtt{grid}}(t) - \mathbb{E}\left[\mathtt{P}_{\mathtt{grid}}(t)\right])\right]$.

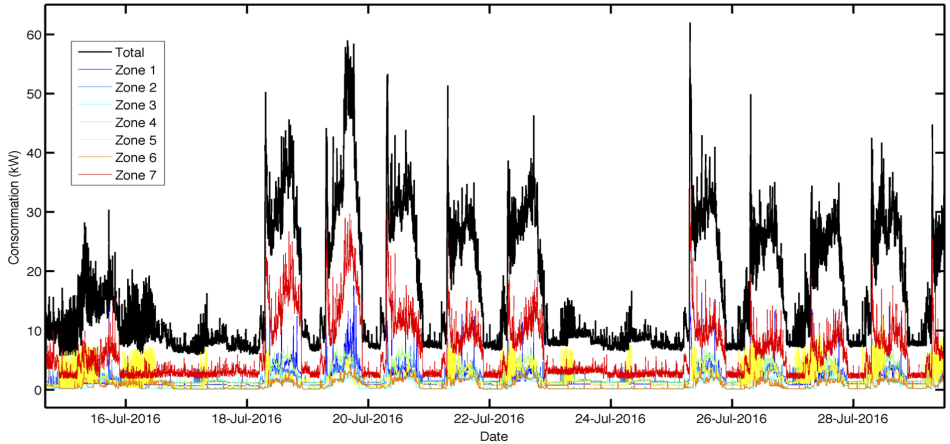
✓ **Optimal stochastic control problem of McKean type** (involving the distribution of State Variables and **of Control**), see **[Carmona-Delarue, AoP 2015, etc]**

2. **Residential building consumption:**

$$\mathtt{P}_{\mathtt{cons}}(t) = \mathtt{P}_{\mathtt{HVAC}}(t) + \mathtt{P}_{\mathtt{Appliance}}(t) + \mathtt{P}_{\mathtt{Lighting}}(t).$$

- ✓ Lighting: automatic mode, depends on the season and the hour of the day. **Negatively correlated to the irradiance [Not yet handled].**
- ✓ HVAC: automatic mode to maintain a inside temperature within a range (e.g. $[19^o\text{C} - 20^o\text{C}]$). **Correlated to the weather conditions [Not yet handled].**
- ✓ Usually modeled with **mean-reverting process with jumps** (when switch off-on devices or start/stop activities).

  Example with a industrial building (tertiary sector):



3. **Power balance:**

$$\mathtt{P}_{\mathtt{cons}}(t) = \mathtt{P}_{\mathtt{bat}}(t) + \mathtt{P}_{\mathtt{sun}}(t) + \mathtt{P}_{\mathtt{grid}}(t)$$

with $\mathtt{P}_{\mathtt{bat}} \gtrless 0$, $\mathtt{P}_{\mathtt{sun}} \geq 0$, $\mathtt{P}_{\mathtt{grid}} \geq 0$ (no selling of extra production).

✓ $\mathtt{P}_{\mathtt{sun}}$: depends on irradiance (see later), humidity, temperature, PV panel...

✓ $\mathtt{P}_{\mathtt{bat}}$: depends on the controller u_t

- $\mathtt{SOC}$: the State Of Charge variable.
- Power delivered by the battery:

$$\mathtt{P}_{\mathtt{bat}}(t) = \phi^{\mathtt{bat}}(u_t, \mathtt{SOC}(t)).$$

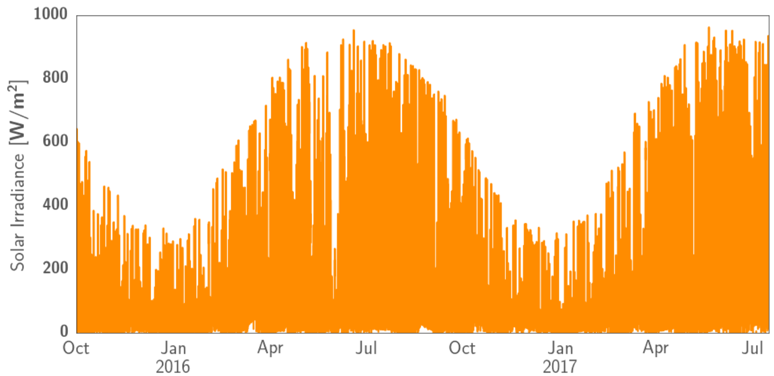
  * u and $\mathtt{P}_{\mathtt{bat}}$ have the same signs
  * if $\mathtt{SOC}(t) = 0$ and $u_t > 0$, no extra discharge ($\mathtt{P}_{\mathtt{bat}}(t) = 0$). And vice-versa.
- Evolution of SOC:

$$\frac{\mathrm{d}\mathtt{SOC}^u(t)}{\mathrm{d}t} = \phi^{\mathtt{SOC}}(u_t, \mathtt{SOC}^u(t)).$$

- Rough approximation: linear dynamics

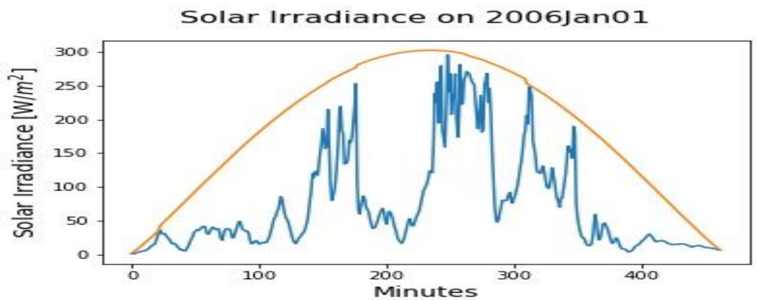
4. Irradiance

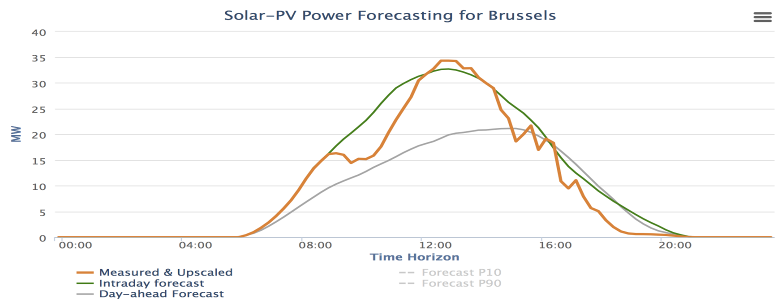
(a) No stationarity property in weather variables



Measurements of Global Horizontal Irradiance from SIRTA (48.7°N, 2.2°E.) for the considered period. Cumulated over 1 day.

(b) Daily fluctuations of irradiance depend much on the location and on the size of the area





Small site Large site

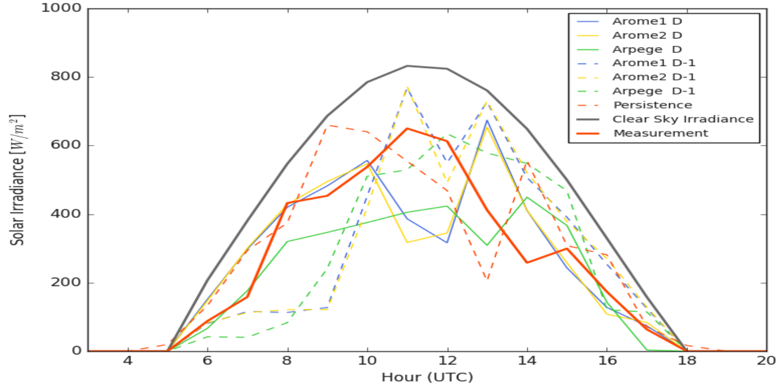
Typically, grid resolution = 1.3km for weather forecast.

(c) Long term forecast are especially difficult (➠ T=1 day)

(d) We need probabilistic forecast ($\neq$ pointwise forecast) at a given location

How to design a stochastic model?

✓ We shall take advantage of day-ahead forecasts (performed on day D-1)



Different MeteoFrance forecasts (AROME, ARPEGE).
Horizons: Day=D, Day before=D-1.

✓ We account for the maximal irradiance (Clear Sky Model = cloudless sky). Good proxy (using the Sun-Earth geometry and the day of the year):

$$\mathbf{I}^{\texttt{clear sky model}}(\mathbf{t}) = [\mathbf{83.69}\ \sin(\frac{\mathbf{2}\pi}{\mathbf{365.24}}(\mathbf{D}+\mathbf{82.07})) + \mathbf{1130.44}]\ \cos(\theta_{\mathbf{z}}(\mathbf{t}))^{\mathbf{1.2}}$$

where D is the day of the year $[0, 365]$, and $\theta_z(t)$ is the solar zenith angle.

✓ **Clear Sky Index**: $X_t = \dfrac{I(t)}{I^{\texttt{clear sky model}}(t)} \in [0, 1]$

✓ **Expected CSI**: $x_t^{\texttt{forecast}} := \dfrac{I^{\texttt{forecast}}(t)}{I^{\texttt{clear sky model}}(t)} \in [0, 1]$

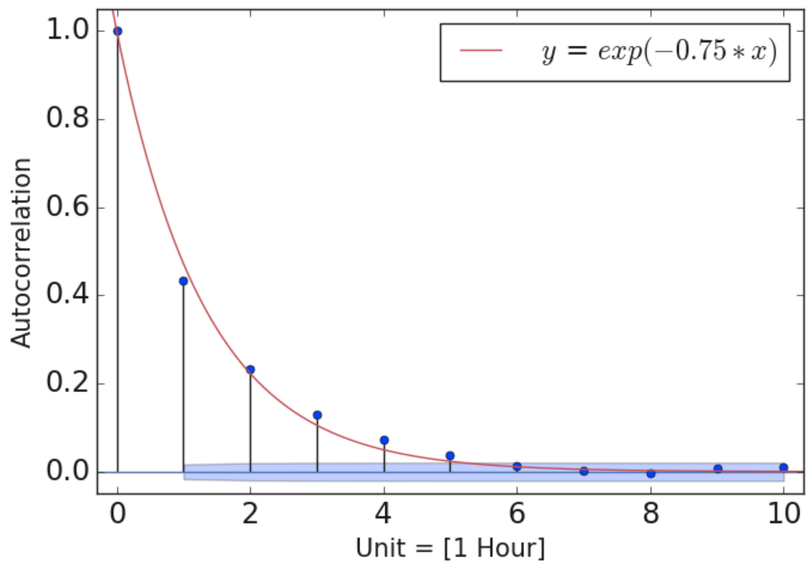
✓ SDE model (like Fisher-Wright or Jacobi process):

$$\mathrm{d}X_t = -a(X_t - x_t^{\texttt{forecast}})\mathrm{d}t + \sigma X_t^{\alpha}(1 - X_t)^{\beta}\mathrm{d}W_t$$

with $\alpha, \beta \in [\frac{1}{2}, 1]$.

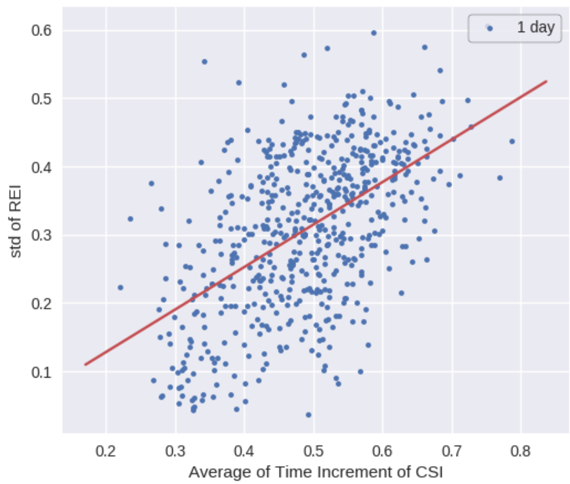
✓ Parameter estimations:

- $\mathbf{a \approx 0.75h^{-1}}$: estimated from autocorrelogram



- $(\alpha, \beta) \approx (\mathbf{0.8}, \mathbf{0.7})$: $\dfrac{\mathbf{X_{t_{i+1}} - X_{t_i}}}{\mathbf{X_{t_i}^{\alpha}(1 - X_{t_i})^{\beta}}} \overset{\mathbf{d}}{\approx} \mathcal{N}(\mathbf{0}, \sigma_{\mathbf{D}}^{\mathbf{2}}\delta)$.
- the **volatility** σ_D is adjusted everyday as a function of the averaged increments of the $D-1$-forecast

$$\mathrm{ATICSI}_D = \sum_{t_i \in \text{Arome forecast times}} \left| \frac{I^{\texttt{forecast}}(t_{i+1})}{I^{\texttt{clear sky model}}(t_{i+1})} - \frac{I^{\texttt{forecast}}(t_i)}{I^{\texttt{clear sky model}}(t_i)} \right|.$$

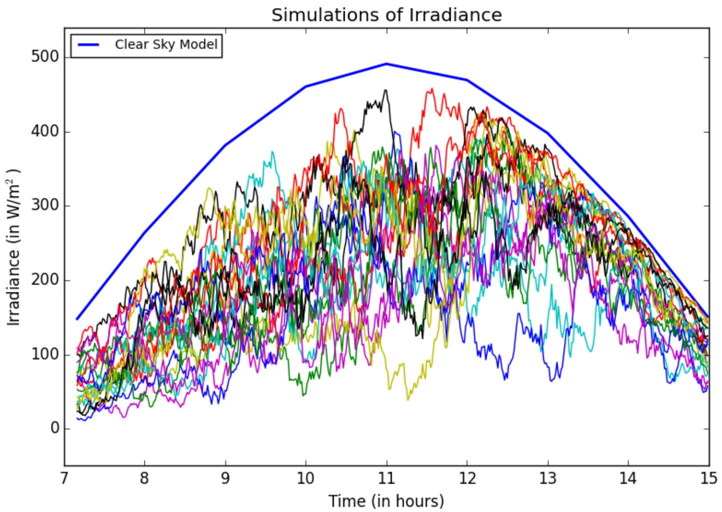


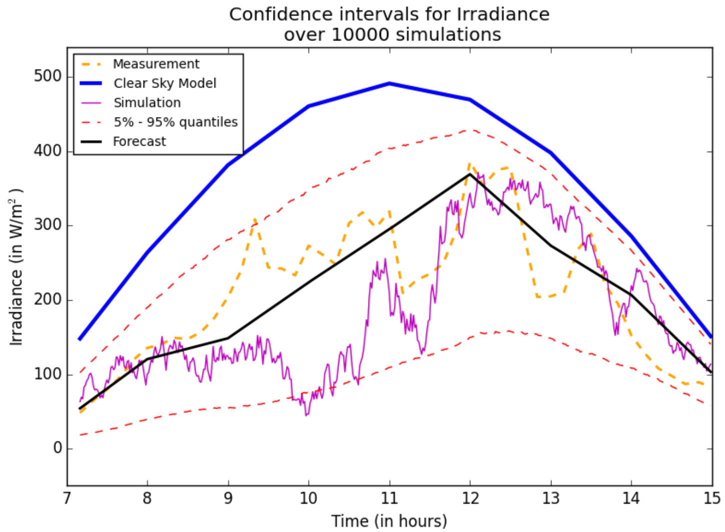
For each day D in the data set, the standard deviation $\sigma_D\sqrt{\delta}$ as a function of the Average Time Increment CSI.

Linear relation of the form

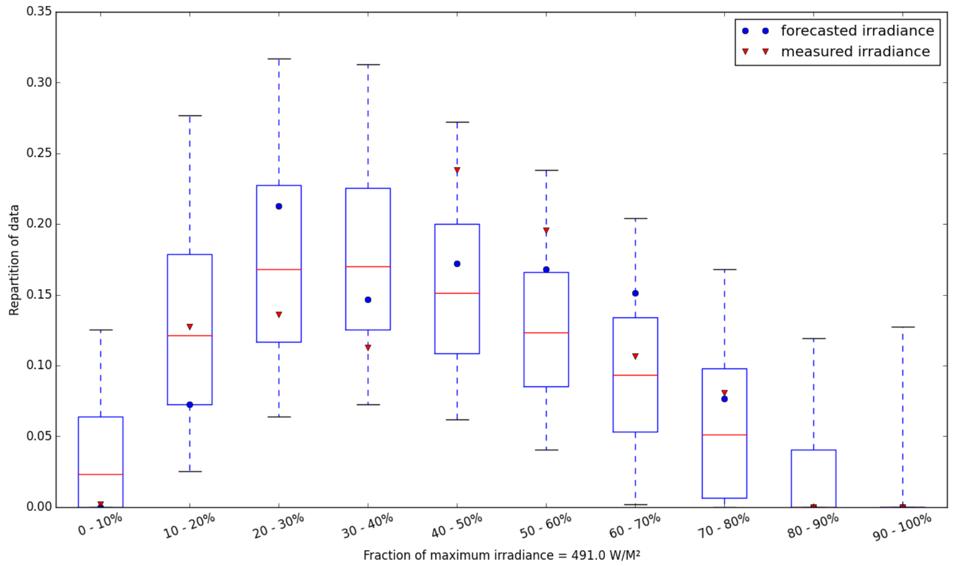
$$\sigma_{\mathbf{D}}\sqrt{\delta} = \mathbf{0.622} \times \mathrm{ATICSI}_{\mathbf{D}} + \mathbf{0.0004} + \text{error}.$$

Results for a day with mitigated weather (October, 24th, 2015)





Results for the probabilistic forecast



Distribution of the irradiance over the day

2 Optimal stochastic control of McKean type

Accounting on the distribution of the system (through its **moments**): set $X_t^u = x_0 + \int_0^t \phi(s, \omega, u_s, X_s^u)\mathrm{d}s + Z_t$ (with Z **exogenous càdlàg process**) and

$$\mathcal{J}(\mathbf{u}) = \mathbb{E}\left[\int_0^T l\left(t, \omega, u_t, X_t^u, \mathbb{E}\left[g(t, \omega, u_t, X_t^u)\right]\right)\mathrm{d}t + \psi(\omega, X_T^u, \mathbb{E}\left[k(\omega, X_T^u)\right]\right] \to \min_{u \text{ pred.}}$$

Standard Lipschitz/differentiability and measurability assumptions on $\phi, \mathbf{l}, \mathbf{g} : [\mathbf{0}, \mathbf{T}] \times \mathbf{\Omega} \times \mathbb{R}^{\mathbf{d}} \times \mathbb{R}^{\mathbf{p}} \times \cdots \mapsto \mathbb{R}^{\cdots}$ and $\psi, \mathbf{k} : \mathbf{\Omega} \times \mathbb{R}^{\mathbf{d}} \times \cdots \mapsto \mathbb{R}^{\cdots}$

References of such a problem (without the distribution on the control): **[Carmona, Delarue, Lachapelle, 2013]**, **[Carmona, Delarue, 2015]** ...

Our strategy of analysis, using Pontryagin principle:

1. necessary conditions by Gateaux differentiability ➠ McKean Forward Backward SDE
2. well-posedness of the McKean FBSDE
3. sufficient conditions under convexity conditions

2.1 Necessary conditions

Theorem (Gâteaux derivatives). Let $u \in \mathbb{H}^2$ and set $\bar{g}_t^u := \mathbb{E}\left[g(t, u_t, X_t^u)\right]$. Assume smooth coefficients, define the FBSDE (Y, M)

$$\begin{cases} -\mathrm{d}Y_t = \Big(\nabla_x \phi(t, u_t, X_t^u) Y_t + \nabla_x l(t, u_t, X_t^u, \bar{g}_t^u) \\ \qquad\qquad + \nabla_x g(t, u_t, X_t^u) \mathbb{E}\left[\nabla_{\bar{g}} l(t, u_t, X_t^u, \bar{g}_t^u)\right] \Big) \mathrm{d}t - \mathrm{d}M_t, \\ Y_T = \nabla_x \psi\left(X_T^u, \mathbb{E}\left[k(X_T^u)\right]\right) + \nabla_x k(X_T^u) \mathbb{E}\left[\nabla_{\bar{k}} \psi\left(X_T^u, \mathbb{E}\left[k(X_T^u)\right]\right)\right] \end{cases}$$

and assume that it has a square integrable solution (Y, M). Then, for any $v \in \mathbb{H}^2$,

$$\partial_\varepsilon \mathcal{J}(\mathbf{u} + \epsilon \mathbf{v})|_{\varepsilon = \mathbf{0}} = \mathbb{E}\left[\int_{\mathbf{0}}^{\mathbf{T}} \Big\{ \mathbf{l_u}(\mathbf{t}, \mathbf{u_t}, \mathbf{X_t^u}, \bar{\mathbf{g}}_\mathbf{t}^\mathbf{u}) + \mathbb{E}\left[\mathbf{l_g}(\mathbf{t}, \mathbf{u_t}, \mathbf{X_t^u}, \bar{\mathbf{g}}_\mathbf{t}^\mathbf{u})\right] \mathbf{g_u}(\mathbf{t}, \mathbf{u_t}, \mathbf{X_t^u}) \\ + \mathbf{Y}_{\mathbf{t}-}^\top \phi_\mathbf{u}(\mathbf{t}, \mathbf{u_t}, \mathbf{X_t^u}) \Big\} \mathbf{v_t} \mathrm{d}\mathbf{t} \right].$$

We allow jumps in the dynamics ➠ cadlag martingale M.

2.2 McKean FBSDE

Theorem (Existence, uniqueness). Under technical assumptions, there is a control u and a McKean-FBSDE (Y, M) satisfying the first-order optimality conditions:

$$\begin{cases} l_u(t, u_t, X_t^u, \bar{g}_t^u) + \mathbb{E}\left[l_g(t, u_t, X_t^u, \bar{g}_t^u)\right] g_u(t, u_t, X_t^u) + (Y_{t-}^u)^\top \phi_u(t, u_t, X_t^u) = 0, \\ -\mathrm{d}Y_t = \Big(\nabla_x \phi(t, u_t, X_t^u) Y_t + \nabla_x l(t, u_t, X_t^u, \bar{g}_t^u) \\ \qquad\qquad + \nabla_x g(t, u_t, X_t^u) \mathbb{E}\left[\nabla_{\bar{g}} l(t, u_t, X_t^u, \bar{g}_t^u)\right]\Big) \mathrm{d}t - \mathrm{d}M_t, \\ Y_T = \nabla_x \psi\left(X_T^u, \mathbb{E}\left[k(X_T^u)\right]\right) + \nabla_x k(X_T^u) \mathbb{E}\left[\nabla_{\bar{k}} \psi\left(X_T^u, \mathbb{E}\left[k(X_T^u)\right]\right)\right]. \end{cases}$$

- ✓ *"Technical assumptions"*:
  - ▸ In general, small coefficients and small time (fixed-point argument)
  - ▸ For linear-quadratic problem, solution in arbitrary time
- ✓ For LQ problems, explicit solution through the solution of Ricatti equations
- ✓ In general, resolution via regression Monte Carlo (like for BSDEs)

2.3 Sufficient conditions

Simplified presentation with $k = 0$. Assume

1. The terminal cost ψ is convex.
2. The mapping

$$\mathcal{H} : \begin{cases} \mathbb{H}^{2,2} \times \mathbb{H}^{\infty,2} \times \mathbb{H}^{\infty,2} & \to \mathbb{R} \\ (u, X, Y) & \mapsto \int_0^T \mathbb{E}\left[l\left(t, u_t, X_t, \mathbb{E}\left[g\left(t, u_t, X_t\right)\right]\right) + Y_{t-}^\top \phi(t, u_t, X_t)\right] \mathrm{d}t \end{cases}$$

   is convex in (u, X) for any Y.

Hamiltonian in expectation and not pathwise.

Theorem. If (u, Y) is the solution of McKean FBSDE, then the control u is optimal.

All conditions are satisfied in the initial microgrid problem.

2.4 Example: the Linear-Quadratic case

Dynamic of the system: $\dfrac{\mathrm{d}\mathtt{SOC}}{\mathrm{d}t}(t) = -\dfrac{\mathtt{P}_{\mathtt{bat}}(t)}{\mathcal{E}_{\max}}.$

- ✓ $\mathtt{SOC}$ (or X): level of charge of the battery
- ✓ $\mathtt{P}_{\mathtt{bat}}$ (or u): power supplied by the battery
- ✓ $\mathcal{E}_{\max}$: energy capacity of the battery
- ✓ $\mathtt{P}_{\mathtt{grid}}(t) = \mathtt{P}_{\mathtt{load}}(t) - \mathtt{P}_{\mathtt{bat}}(t)$: power balance

Optimization criterion:

$$\min_{\mathtt{P}_{\mathtt{bat}}(.)} \int_0^T \Big(\kappa_t \, \mathbb{V}\mathrm{ar}(\mathtt{P}_{\mathtt{grid}}(t)) + \lambda_t \, \mathbb{E}\left[\mathtt{P}^2_{\mathtt{grid}}(t)\right] + \mu_t \, \mathbb{E}\left[\mathtt{P}^2_{\mathtt{bat}}(t)\right]\Big) \, \mathrm{d}t$$
$$+ \int_0^T \Big(\nu_t \, \mathbb{E}\left[\left(\mathtt{SOC}(t) - \mathrm{soc}_t\right)^2\right]\Big) \, \mathrm{d}t + \frac{\omega}{2} \, \mathbb{E}\left[\left(\mathtt{SOC}(T) - \mathrm{soc}_T\right)^2\right]$$

Assumptions: $\kappa, \lambda, \mu, \omega \geq 0$ and $\lambda + \mu > 0$

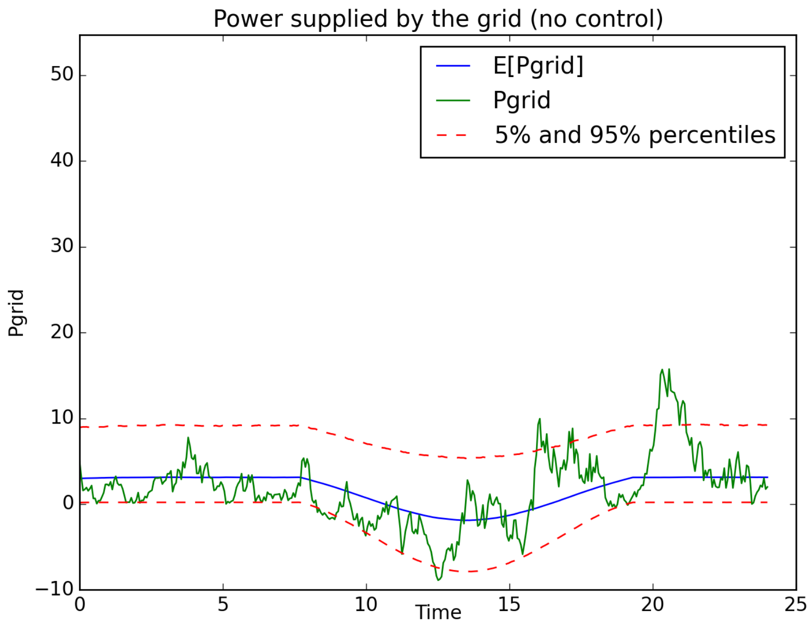
McKean FBSDE:

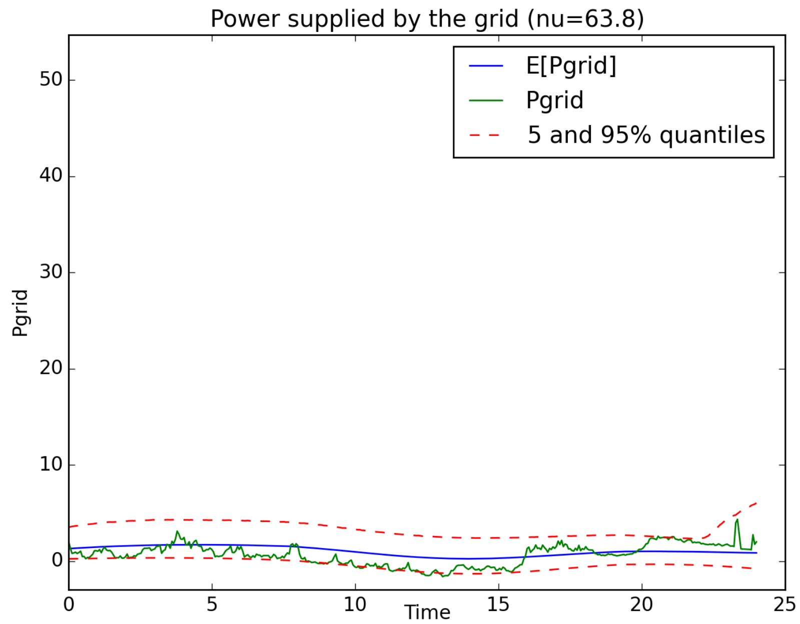
$$\begin{cases} \mathrm{d}X_t = -\frac{u_t}{\mathcal{E}_{\max}}\,\mathrm{d}t, \\ X_0 = \mathtt{SOC}(0), \\ -\mathrm{d}Y_t = \nu_t\,\left(X_t - \mathrm{soc}_t\right)\,\mathrm{d}t - \mathrm{d}M_t, \\ Y_T = \omega\left(X_T - \mathrm{soc}_T\right), \\ \left(\kappa_t + \lambda_t + \mu_t\right)u_t = \left(\kappa_t + \lambda_t\right)\mathtt{P}_{\mathtt{load}}(t) + \kappa_t\,\mathbb{E}\left[u_t - \mathtt{P}_{\mathtt{load}}(t)\right] + \frac{Y_t}{\mathcal{E}_{\max}}. \end{cases}$$

➠ Closed-form solution (Ricatti equations).

2.5 Numerical illustration: with or without battery command

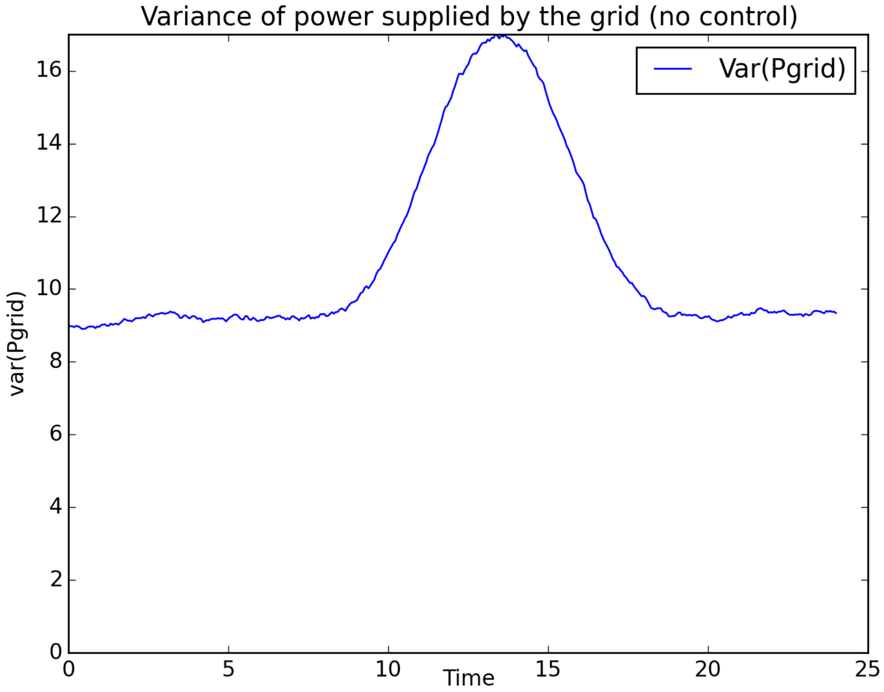
Here we consider the Linear-Quadratic case (explicit solution).



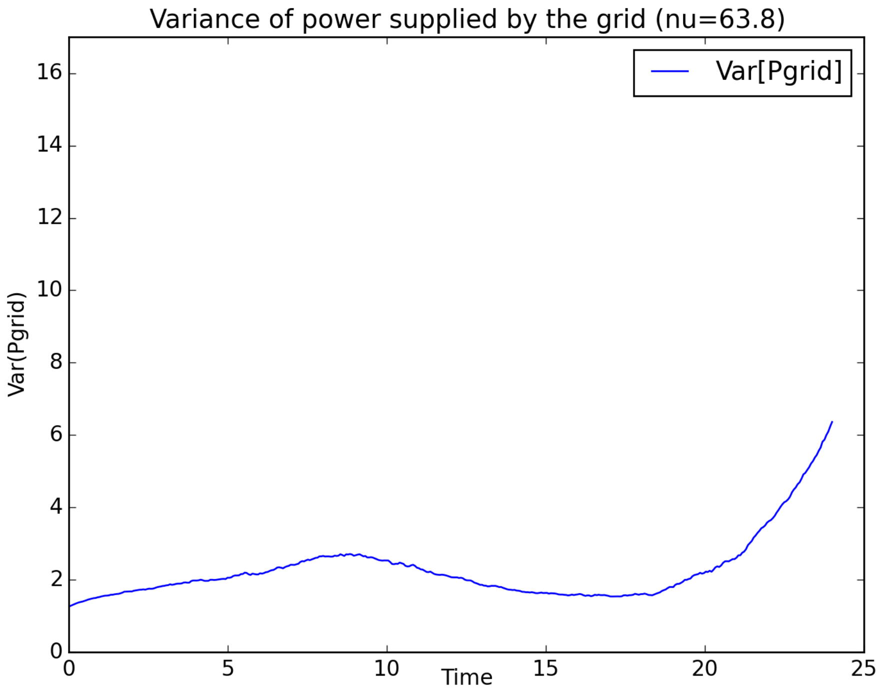


WITHOUT battery WITH battery

Pathwise behavior and Distribution of P_{grid}

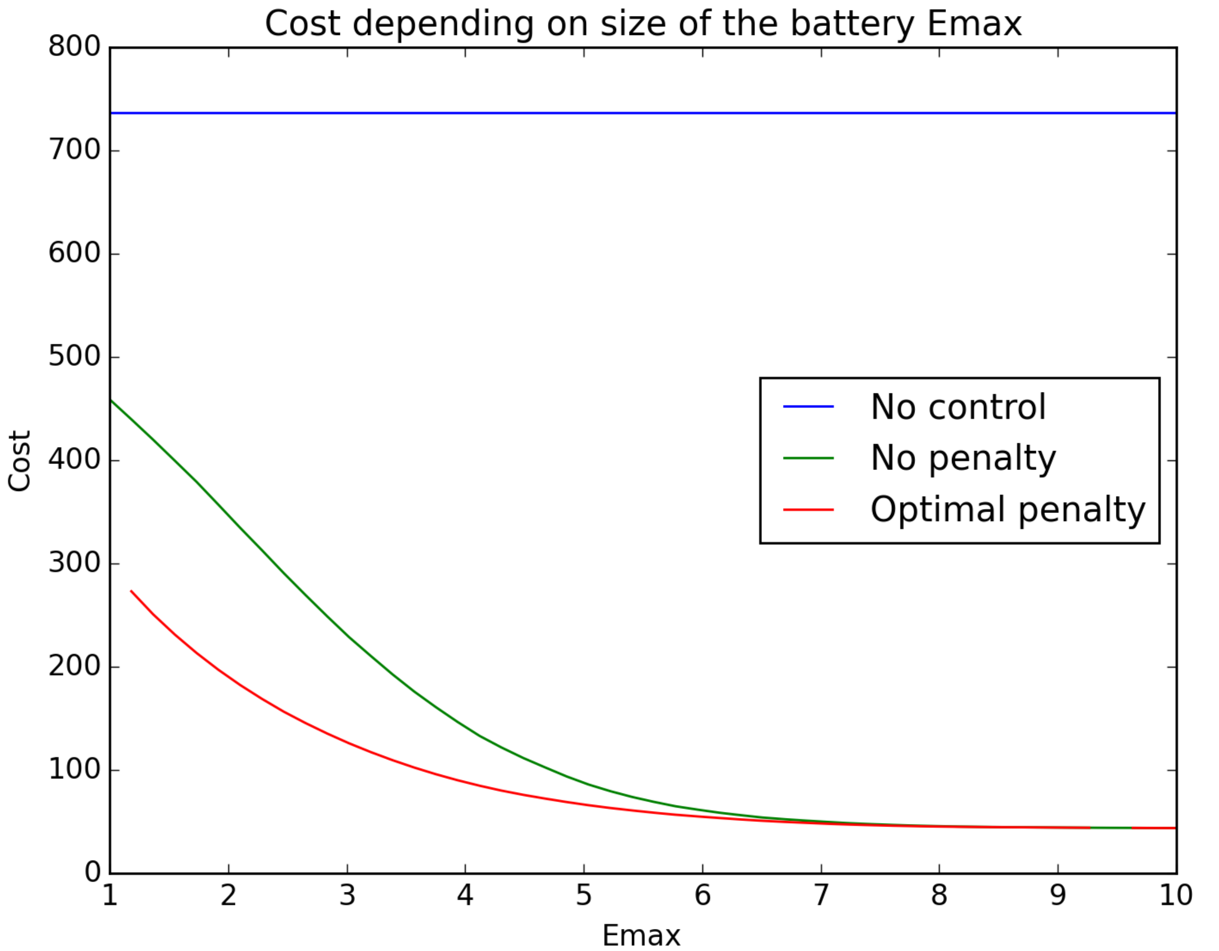


WITHOUT battery



WITH battery

Empirical Variance of P_{grid}



Impact of the size of the battery on the cost and the optimal penalization parameter

3 Conclusion

✓ Modeling micro-grid management

- Optimization criterion: variability of $\mathtt{P}_{\mathtt{grid}}$
- New irradiance modeling: using SDE. Good probabilistic forecast
- Optimal control: solution by Pontryagin principle, and McKean FBSDE

✓ Perspectives:

- Numerical resolution in general: design of Regression Monte-Carlo
- Coupling consumption to weather variables: Lighting $\longleftrightarrow$ irradiance, inside temperature $\longleftrightarrow$ outside temperature and irradiance ...
- Coupling with wind farms

✓ Questions:

- Cost of installation (battery aging) vs savings using the management system
- Other storage capacities (heat networks, flywheel. . .) [Maxime Grangereau PhD thesis with EDF]
- Individual storage capacity vs mutualized ones?
- Which size for aggregating production/consumption?
- Impact of time-inconsistency

Thank you for your attention!