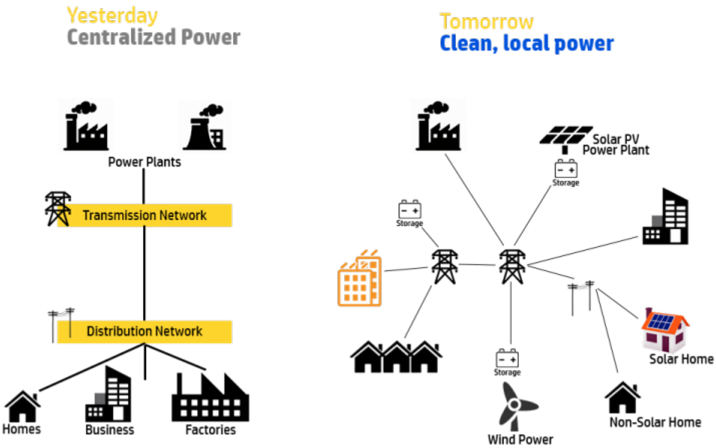
Stylized model for a grid with distributed generation and storage

Alasseur Clémence [1]   Ben Tahar, Imen [2]   Matoussi, Anis [3]

CAESARS 2018
Advances in Modelling and Control for Power Systems of the Future

[1] EDF R&D and Finance for energy Market Research Centre (FiME)
[2] CEREMADE and Finance for energy Market Research Centre (FiME)
[3] Institut du Risque et de l'Assurance, Le Mans Université


Yesterday
Centralized Power
Power Plants
Transmission Network
Distribution Network
Homes
Business
Factories
Tomorrow
Clean, local power
Solar PV
Power Plant
Storage
Storage
Storage
Solar Home
Wind Power
Non-Solar Home

Context

- Transition from a centralized vertically-integrated power systems to a **new** scheme with small scale distributed generation and storage.
- Projected increase in production of renewables, the share of solar and wind will continue to increase ...
- Electricity storage triggered by the expansion of geographically (large to small scale) distributed generation

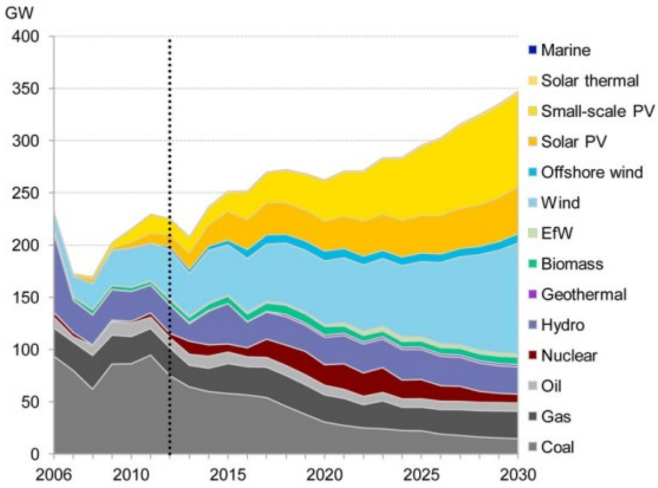


Figure: Projections for Power Generation Capacity to 2030

source: Bloomberg New Energy Finance's global forecast

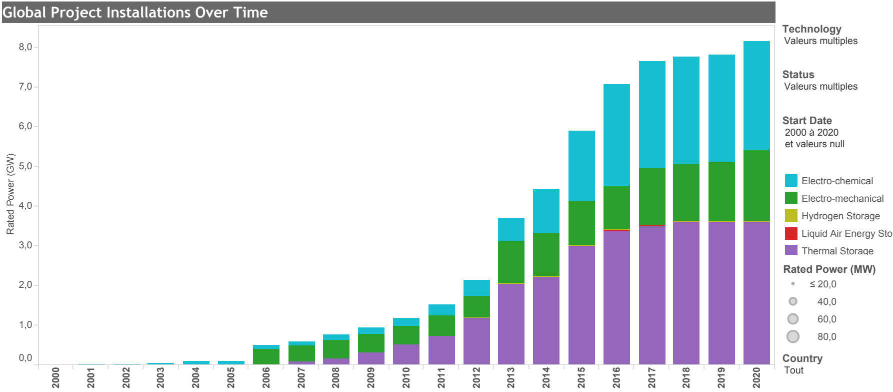


Figure: Projected evolution for Storage Installation (⊖ Pumped Hydro)
source: DOE Global Energy Storage database

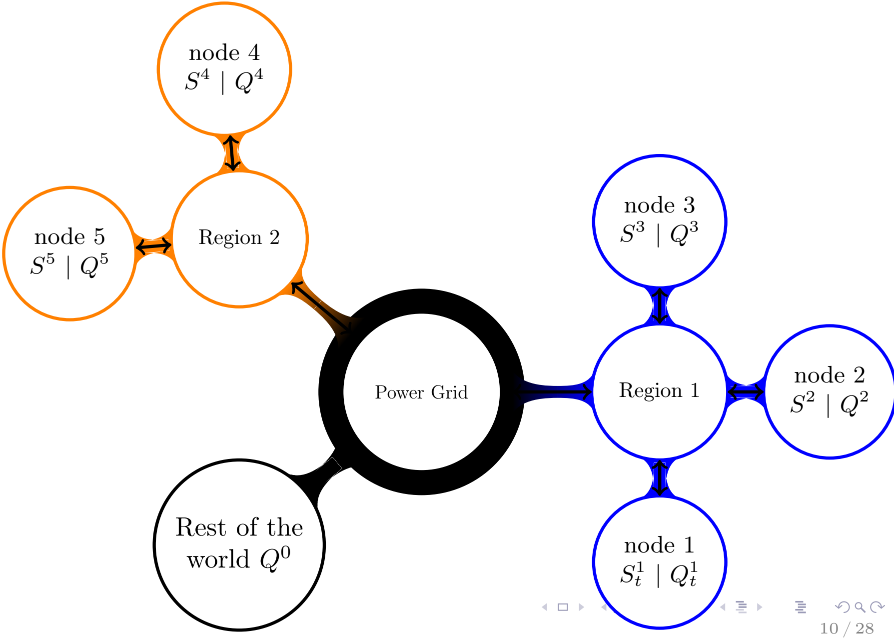
Aim of this work:

- provide a stylized quantitative model for a power grid with distributed generation and storage
- tractable model to assess some questions arising in this grid:
    - optimal storage sizing and management,
    - centralized versus decentralized management of the system,
    - impact of energy and distribution tariff structure,
    - ...

- Growing literature on distributed storage management,
- Quantitive approaches:
  Couillet et. al (2012) Electrical Vehicles in the smart grid: a mean field game analysis.
  De Paola et. al. (2016) Distributed control of micro-storage devices with mean field games.

- Stylized model for the power grid with N nodes
- Extended Mean Field Game approximation
- Characterization in a tractable case
- Some numerical simulations

Stylized model for the Power Grid



Grid $=N$ nodes, Γ regions ; state variables of node i in region γ.

- control of node i is the storage rate $\alpha^i \in \mathcal{A}$
- $Q^{i,\gamma} - \alpha_t^i =$ power [injection if ≥ 0] / [consumption if ≤ 0]

$$Q_t^{i,\gamma} = Q_0^{i,\gamma} + \int_0^t \mu^\gamma(t)dt + \int_0^t \sigma_t^\gamma dB_t^i + \int_0^t \sigma_t^{\gamma,0} dB_t^0, \qquad (1)$$

$$S_t^{i,\gamma} = S_0^{i,\gamma} + \int_0^t \alpha_u^i du, \quad \text{energy in storage device} \qquad (2)$$

- $B^0, B^1, \cdots, B^N$ independent Brownian motions on $(\Omega, \mathcal{F}, \mathbb{P}; \mathbb{F})$
- $\mathbb{F}^0 := \{\sigma(B_s^0),\ s \leq t\}_{t \in \mathbb{R}_+}$ is the filtration of common noise B^0

When the strategy $\alpha = (\alpha^1, \cdots, \alpha^N) \in \mathcal{A}^N$ is implemented:

Electricity Price

$$P_t^{N,\alpha} \quad = \quad p\left(-Q_t^0 - \sum_{i=1}^{N} \eta(Q_t^i - \alpha_t^i)\right),$$

- $p(\cdot)$ exogenous inverse demand function,
- η is a scaling parameter, $\eta = 1/N$

Assumption

The function $p(\cdot)$ is assumed to be strictly increasing.

When the strategy $\alpha = (\alpha^1, \cdots, \alpha^N) \in \mathcal{A}^N$ is implemented:

Cost Structure

$$J^{i,\gamma,N}(\alpha) = \underbrace{\mathbb{E}\left[\int_0^T P_t^{N,\alpha}\left(\alpha_t^i - Q_t^i\right) dt\right]}_{\text{volumetric charge}} + \underbrace{\mathbb{E}\left[\int_0^T L_T^{\gamma}(Q_t^i, \alpha_t^i) dt\right]}_{\text{demand charge}}$$

$$+ \underbrace{\mathbb{E}\left[\int_0^T L_S(S_t^{i,\alpha^i}, \alpha_t^i) dt + g(S_T^{i,\alpha^i})\right]}_{\text{storage cost}}.$$

$$J^{0,N}(\alpha) = \underbrace{\mathbb{E}\left[\int_0^T -P_t^{N,\alpha} Q_t^0 dt\right]}_{\text{energy cost}} + \underbrace{\mathbb{E}\left[\int_0^T L_T^0(Q_t^0, 0) dt\right]}_{\text{transmission cost}} \tag{3}$$

Assumption

The current cost $(s, q, \alpha) \mapsto L_T^\gamma(q, \alpha) + L_S(s, \alpha)$ *is strictly convex with respect to* (s, α). *The terminal cost* $s \mapsto g(s)$ *is strictly convex with respect to* s.

Assumption

There exists some constant $C > 0$ *such that*

$$\begin{aligned} \frac{1}{C}\left(|q|^2 + |s|^2 + |a|^2\right) - C \quad &\leq \quad L_T^\gamma(q, a) + L_S(s, a) + g(s) \\ &\leq C\left(|q|^2 + |s|^2 + |a|^2\right) + C. \end{aligned}$$

Assumption

The functions L_T^γ, L_S *and* g *are continuously differentiable.*

Optimality criterion

1. **Non-cooperative game point of view**: We are led to the analysis of a non-zero sum stochastic game with N players and to the search of Nash-equilibria.
2. **Central planner point of view**: We consider the power grid model from the perspective of a central planner whose aim is to dictate a storage rule: $\alpha = (\alpha^1, \cdots, \alpha^N)$ in order to minimize the *egalitarian* cost function between the nodes and the rest of the world

$$J^{\mathrm{C,N}}(\alpha) = J^{r,N}(\alpha) + \sum_{i=1}^{N} \eta J^{i,\gamma,N}(\alpha).$$

For the sake of simplicity $\Gamma = 1$ region.

- **Notation**: for an $\mathbb{F}$-adapted process $\xi = \{\xi_t\}$, denote: $\bar{\xi}_t := \mathbb{E}[\xi_t | \mathcal{F}_t^0]$
- **Initial state**: $x_0 = (s_0, q_0^0, q_0)$ random vector independent from $\mathbb{F}^0$.
- **Admissible control processes**: $\mathcal{A}$ the set of $\mathbb{F}$-adapted real-valued processes $a = \{a_t\}$ such that $\mathbb{E}\left[\int_0^T |a_u|^2 du\right] < \infty$.
- **State processes**:

$$
\begin{aligned}
Q_t^\gamma &= q_0^\gamma + \int_0^t \mu^\gamma(u, Q^\gamma) du + \int_0^t \sigma^\gamma(u, Q^\gamma) dB_u^\gamma + \int_0^t \sigma^{\gamma,0}(u, Q^\gamma) dB_u^0 \\
Q_t^0 &= q_0^0 + \int_0^t \mu^r(u, Q_t^0) du + \int_0^t \sigma^0(u, Q_u^0) dB_u^0 \, .
\end{aligned}
$$

and to an admissible control process $\alpha = \{\alpha_t\} \in \mathcal{A}$

$$
S_t^\alpha = s_0 + \int_0^t \alpha_u du,
$$

1. Non-cooperative game perspective

- a representative agent i chooses the control $\alpha \in \mathcal{A}$
- other players actions approximated by : $\sum_{j \neq i} \alpha_t^j / N \approx \bar{\nu}_t$

$$p\left(-Q_t^0 - \sum_{j=1}^{N} \frac{(Q_t^j - \alpha_t^j)}{N}\right) \quad \approx \quad P_t^{\bar{\nu}} = p\left(-Q_t^0 - \left(\mathbb{E}[Q_t | \mathcal{F}_t^0] - \bar{\nu}_t\right)\right)$$

- Fix an $\mathbb{F}^0$-adapted process $\bar{\nu} = \{\bar{\nu}_t\} \rightarrow$ the cost function

$$J_{x_0}(\alpha, \bar{\nu}) \quad = \quad \mathbb{E} \int_0^T \left[P_t^{\bar{\nu}}(\alpha_t - Q_t) + L_T(Q_t, \alpha_t) + L_S(S_t, \alpha_t)\right] dt + \mathbb{E}\left[g(S_t)\right]$$

Definition (Mean field Nash equilibrium)

$\alpha^\star$ is an MFG equilibrium it minimizes: $\alpha \mapsto J_{x_0}(\alpha, \{\mathbb{E}[\alpha_t^\star | \mathcal{F}_t^0]\})$.

2. Central planner perspective

- central planner dictates a control $\alpha \in \mathcal{A}$
- let $\bar{\alpha}_t = \mathbb{E}[\alpha_t|\mathcal{F}_t^0]$, and $P_t^{\bar{\alpha}} = p\left(-Q_t^0 - \left(\mathbb{E}[Q_t|\mathcal{F}_t^0] - \bar{\alpha}_t\right)\right)$
- consider the cost function

$$J_{x_0}^C(\alpha) \quad = \quad \mathbb{E}\int_0^T \left[P_t^{\bar{\alpha}} Q_t^0 + L_T^0(Q_t^0, 0)\right] dt + J_{x_0}(\alpha^\gamma, \bar{\alpha}_t)$$

Definition (Mean Field Optimal Control)

$\hat{\alpha}$ is an MFC if it minimizes : $\alpha \mapsto J_{x_0}^C(\alpha)$.

1. Stylized model for the Power Grid
2. An Extended Men field Game approximation $N \to \infty$
3. Characterization via coupled systems of FBSDE
4. Simulations in a tractable case

Some References:

- Probabilistic approach: R. Carmona and F. Delarue (2013), (2015); R. Carmona and F. Delarue and D. Lacker (2014).
- Linear Quadratic case: A. Bensoussan, K. Sung, S. Yam and Yung (2011), Yong (2013), Pham (2016).
- Extended MFG in the linear Quadratic case: Graber (2016).

Characterization of MFG equilibria

- For any $\mathbb{F}^0$-adapted process $\bar{\nu}$, there exists a unique control $\alpha^\star(\bar{\nu}, x_0)$ which minimizes the function $J_{x_0}(\alpha, \bar{\nu})$
- Euler optimality condition: $\alpha^\star$ is optimal if and only if there exists a unique adapted solution $(S^\star, Y^\star, Z^{0,\star}, Z^\star)$ of the FBDSE

$$\begin{cases} dS_t^\star &= \alpha_t^\star dt, \quad S_0^\star = s_0 \\ dY_t^\star &= -\partial_s L_S(S_t^\star, \alpha_t^\star) dt + Z_t^{0,\star} dB_t^0 + Z_t^\star dB_t \\ Y_T^\star &= \partial_s g(S_T^\star) \end{cases} \tag{4}$$

satisfying the coupling condition

$$0 = Y_t^\star + \partial_\alpha L_T(Q_t, \alpha_t^\star) + \partial_\alpha L_S(S_t^\star, \alpha_t^\star) + P_t^{\bar{\nu}} \tag{5}$$

- If in addition: $\mathbb{E}\left[\alpha_t^\star | \mathcal{F}_t^0\right] = \bar{\nu}_t$ then $\alpha^\star$ is an MFG equilibrium.

Characterization of MFC

- If $\hat{\alpha}$ minimizes $J^C_{x_0}$ *(central planner cost)* then there exists a unique adapted solution $(\hat{S}, \hat{Y}, \hat{Z}^0, \hat{Z})$ of the FBDSE

$$\begin{cases} d\hat{S}_t &= \hat{\alpha}_t dt, \quad \hat{S}_0 = s_0 \\ d\hat{Y}_t &= -\partial_s L_S(\hat{S}_t, \hat{\alpha}_t)dt + \hat{Z}^0_t dB^0_t + \hat{Z}_t dB_t \\ \hat{Y}_T &= \partial_s g(\hat{S}_T) \end{cases} \tag{6}$$

satisfying the coupling condition

$$\begin{aligned} 0 \quad = \quad & \hat{Y}_t + \partial_\alpha L_T(Q_t, \hat{\alpha}_t) + \partial_\alpha L_S(\hat{S}_t, \hat{\alpha}_t) + P^{\bar{\hat{\alpha}}}_t \\ & -p'\left(-Q^0_t - (\bar{Q}_t - \bar{\hat{\alpha}}_t)\right)\left(Q^0_t + (\bar{Q}_t - \bar{\hat{\alpha}}_t)\right) \end{aligned} \tag{7}$$

Central planner versus Non-cooperative game

Proposition

Assume that $\hat{\alpha}$ is an MFC for the problem with a pricing rule p. Then $\hat{\alpha}$ is an equilibrium for the MFG problem with pricing rule

$$p^{\mathrm{MFG}}(x) \quad = \quad p(x) + xp'(x)\ .$$

ε Nash equilibrium for the N-players game

- let $x_0^i = (s_0^i, q_0^i),\ i = 1, \cdots, N$ be independent initial conditions
- denote by $\alpha^{i,\star}$ the MFG equilibrium associated to x_0^i and B^i

Proposition

For each $\varepsilon > 0$, $\exists N_\varepsilon$ s.t. for $N \geq N_\varepsilon$, for all $i \in \{1, \cdots, N\}$

$$J^{i,N}(\alpha^i, \{\alpha^{j,\star}\}_{j\neq i}) \quad \geq \quad J^{i,N}(\alpha^{1,\star}, \cdots, \alpha^{N,\star}) - \varepsilon, \quad \forall \alpha \in \mathcal{A}$$

Solution in the tractable linear quadratic case

- the pricing rule is linear: $p: \ x \mapsto p_0 + p_1 x$.
- quadratic storage and demand charge costs:

$$\begin{aligned} L_S : \quad & (s, \alpha) \mapsto \frac{A_2}{2} s^2 + A_1 s + \frac{C}{2} \alpha^2 \\ L_T : \quad & (q, \alpha) \mapsto \frac{K}{2} (q - \alpha)^2 \\ g : \quad & s \mapsto \frac{B_2}{2} \left(s - \frac{B_1}{B_2} \right)^2 \end{aligned}$$

where A_1, A_2, B_1, B_2, C and K are some given positive constants.

- Optimization horizon $T \sim 1$ day.
- Random consumptions: the sum of a deterministic seasonal function μ and an Ornstein-Uhlenbeck (OU) process

$$\begin{aligned} dQ_t^i &= -a^\gamma(Q_t^i - \mu^\gamma(t))dt + \sigma^\gamma dB_t^i + \sigma^{\gamma\ 0} dB_t^0\ , \quad Q_0^i = q_0^i, \quad i \in \gamma, \\ dQ_t^0 &= -a^0(Q_t^0 - \mu^0(t))dt + \sigma^0 dB_t^0\ , \quad Q_0^0 = q_0^0. \end{aligned}$$

1. Illustrate some stylized behaviors of the model
   - Management of the storage w.r.t. the bill structure:

$$\begin{aligned} \text{bill structure} &= \text{volumetric part} + \text{demand charge} \\ &= P_t^{N,\alpha}(\alpha_t^i - Q_t^i) + (K/2)(\alpha_t^i - Q_t^i)^2 \end{aligned}$$

   - Dimensioning of the batteries
   - Impact on electricity spot price
2. comparing decentralized versus centralized management
   - evaluating the Price of Anarchy (PoA) ratio = (Total Cost MFG)/(Total cost MFC)

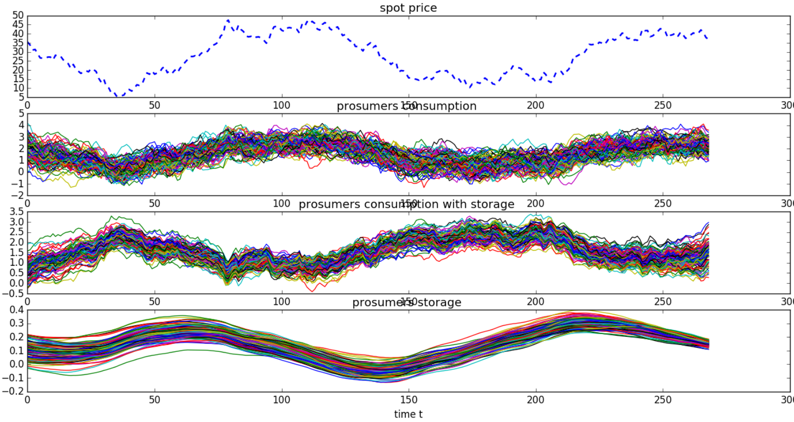


Figure: One simulation of spot price (upper graph), prosumers' consumption $-Q^i$ (middle graph), prosumer's net consumption $-Q^i + \alpha^i$ (lower middle graph) and prosumer' storage level (lower graph) for every prosumers.

- Stylized model for a power grid with decentralized generation and storage
- Tractable setting allowing for easy to implement numerical simulations
- Calibration on real figures ?